Math 126 - Spring 2020 - Exam #1

Name: Answer Key

ID# ______________________

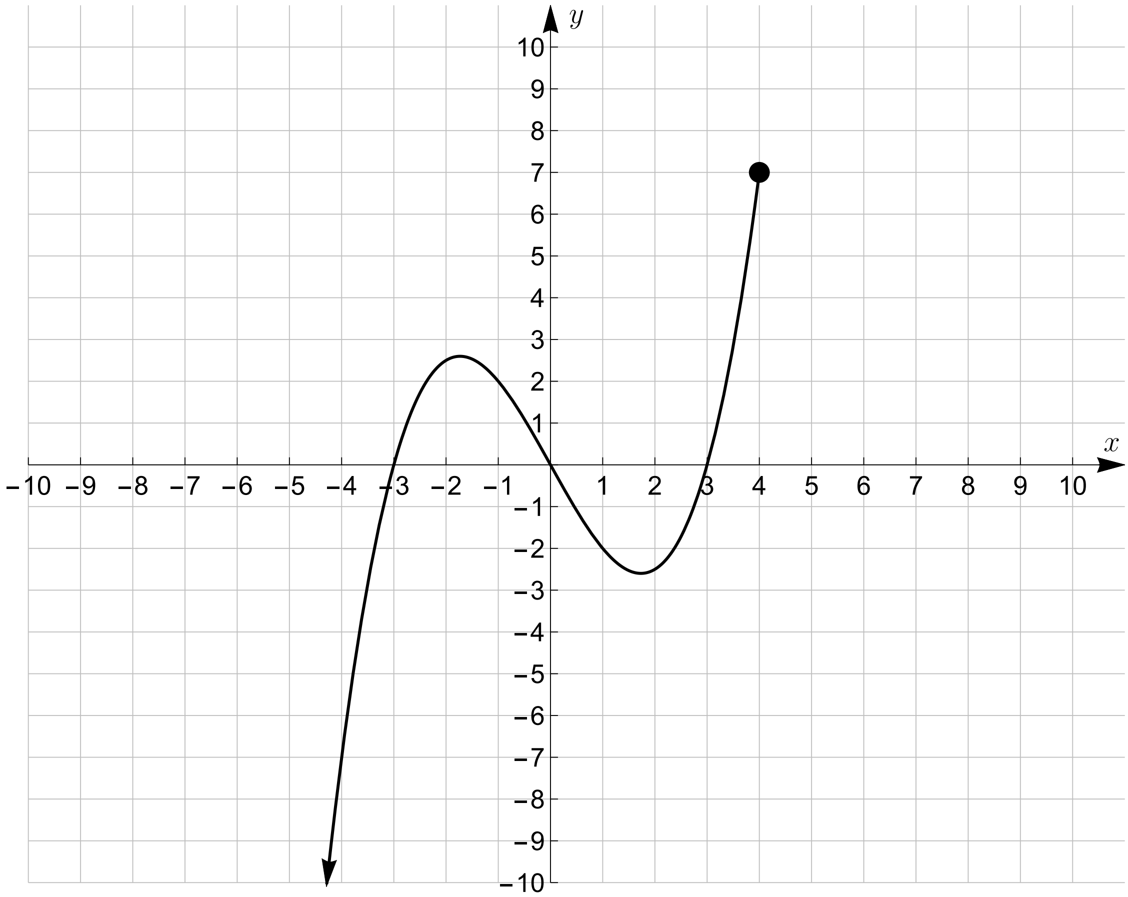
HONOR CODE: On my honor, I have neither given nor received any aid on this examination.

Signature: ______________________

Instructions: Do all scratch work on the test itself. Make sure your final answers are clearly labeled. There are extra blank graphs at the end of the test, in case you need them. If you do use the extra blank graphs at the end of the test, be sure to (1) indicate on the question that you have more work on the extra blank graphs and (2) label your work on the extra blank graphs so I know what work goes with which question. **SHOW ALL WORK ON THIS EXAM IN ORDER TO RECEIVE FULL CREDIT!!!**

No.	Score
1	/6
2	/13
3	/9
4	/10
5	/10
6	/10
7	/12
8	/8
9	/12
10	/10
Total	/100

1. Use the following graph to answer parts (a) and (b).



(a) Determine whether or not the relation graphed above represents a function. **Explain your answer!** (2 points)

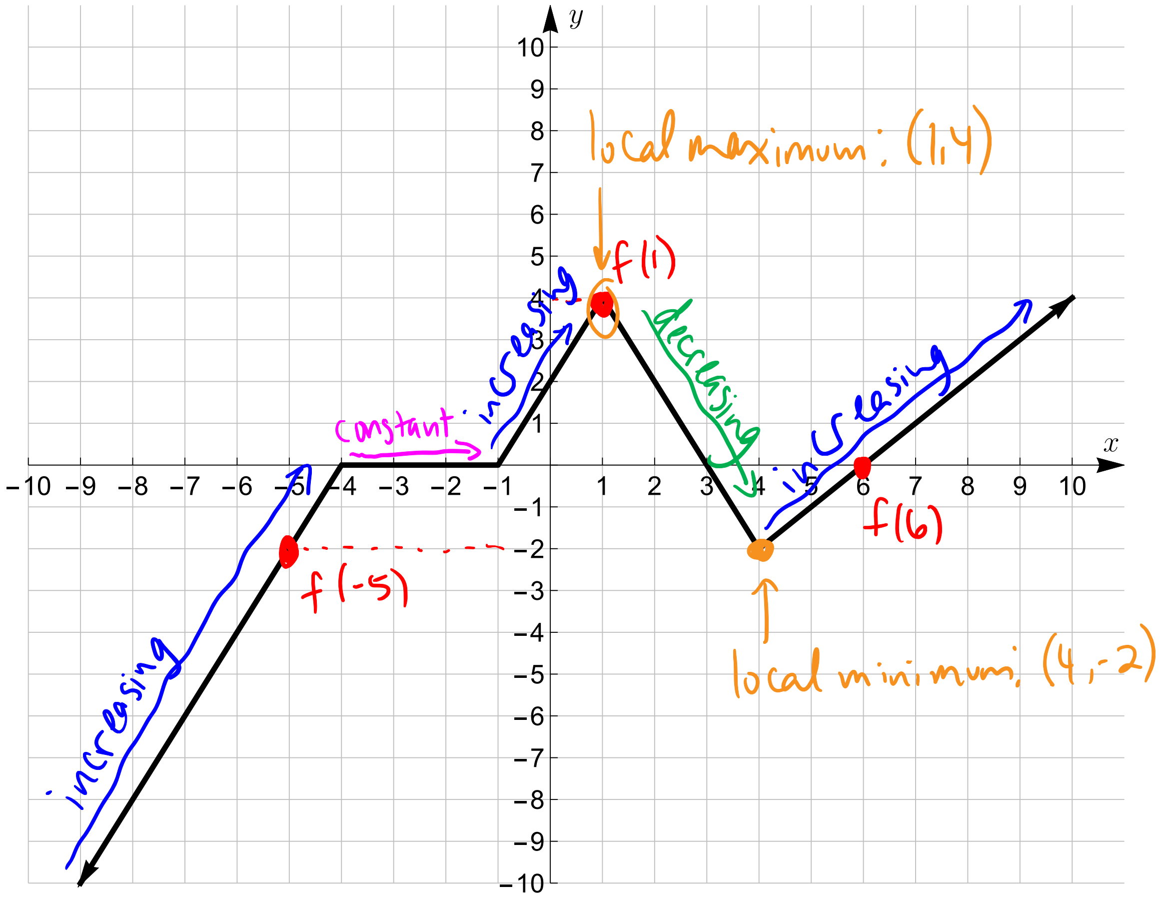
Yes because it's not possible to draw any vertical line that hits the graph in more than one location, so it passes the Vertical Line Test.

(b) State the domain and range of the relation graphed above. (4 points)

Domain $= (-\infty, 4]$ or $\{x \mid x \le 4\}$

Range $= (-\infty, 7]$ or $\{y \mid y \le 7\}$

2. Use the following graph to answer parts (a) - (c).



(a) Find the values of $f(-5)$, $f(1)$, and $f(6)$. (3 points)

$f(-5) = -2$ $\quad$ $f(6) = 0$

$f(1) = 4$

(b) State the intervals on which the function is increasing, decreasing, and constant. (6 points)

increasing: $(-\infty, -4) \cup (-1, 1) \cup (4, \infty)$

decreasing: $(1, 4)$

Constant: $(-4, -1)$

(c) Determine where the local maxima and minima occur and determine what the local maximum and local minimum values are. (4 points)

local maximum at $x=1$ whose value is $f(1)=4$

local minimum at $x=4$ whose value is $f(4)=-2$

3. Determine whether each of the following functions is even, odd, or neither. **Explain your answer!** (3 points each)

(a) $f(x) = \frac{x^4}{2x^3 - x}$

$$f(-x) = \frac{(-x)^4}{2(-x)^3 - (-x)} = \frac{x^4}{-2x^3 + x} = \frac{x^4}{-(2x^3 - x)}$$

$$= -\frac{x^4}{2x^3 - x} = -f(x)$$

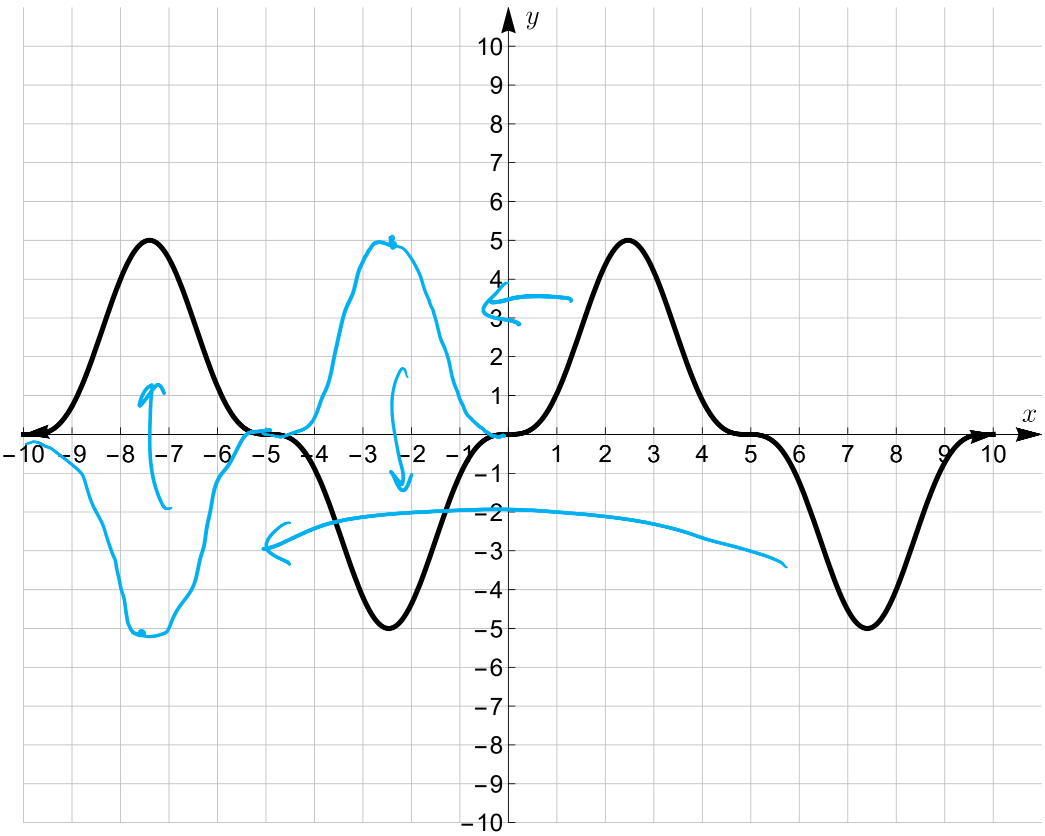
$\boxed{\text{odd because } f(-x) = -f(x)}$

(b) $g(x) = 2x^3 + 3x^2$

$$g(-x) = 2(-x)^3 + 3(-x)^2 = \underbrace{-2x^3 + 3x^2}_{\text{not } g(x)} = \underbrace{-(2x^3 - 3x^2)}_{\text{not } -g(x)}$$

Neither because $g(-x)$ isn't equal to $g(x)$ or $-g(x)$

(c)



odd because it's symmetric about the origin (like flipping about the y-axis then flipping about the x-axis)

4. Let $f(x)$ be the piecewise defined function

$$f(x) = \begin{cases} \frac{1}{2}x + 2 & \text{if } x < 0 \\ -2 & \text{if } x = 0 \\ x^2 & \text{if } 0 < x \leq 3 \end{cases}$$

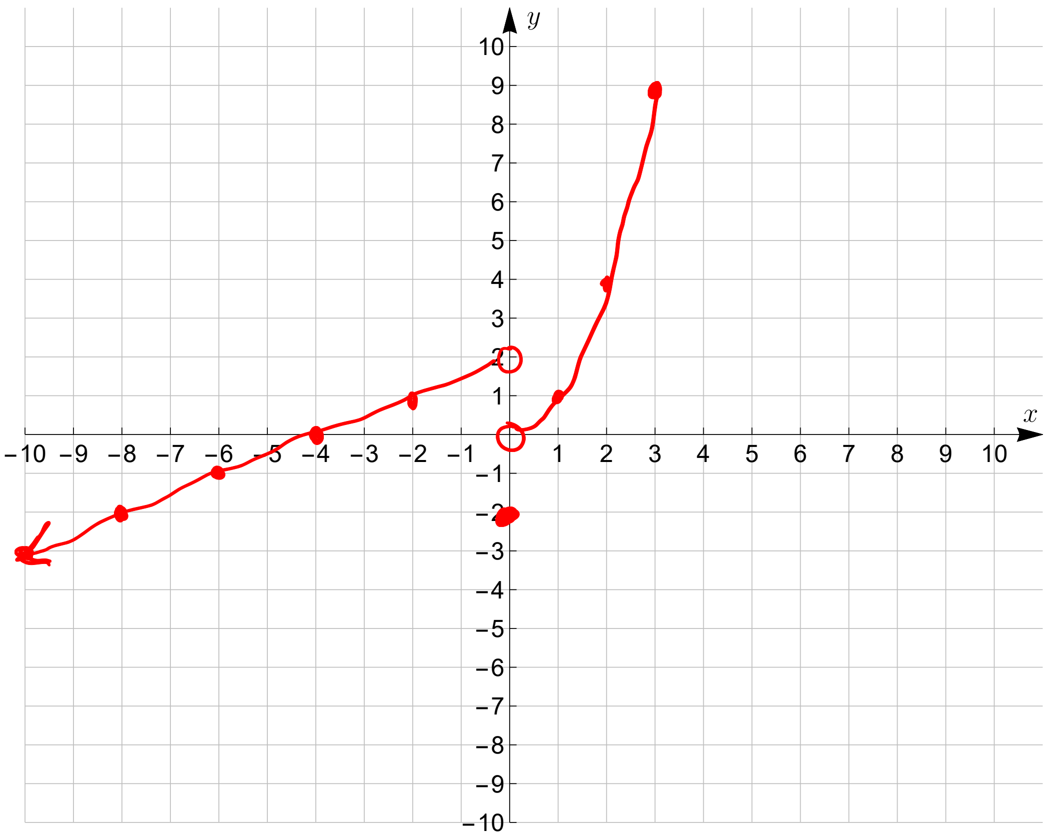
(a) Find $f(3)$. (2 points)

$f(3) = 3^2 = 9$

(b) Find $f(-2)$. (2 points)

$f(-2) = \frac{1}{2}(-2) + 2 = -1 + 2 = 1$

(c) Sketch the graph of $f(x)$. (6 points)



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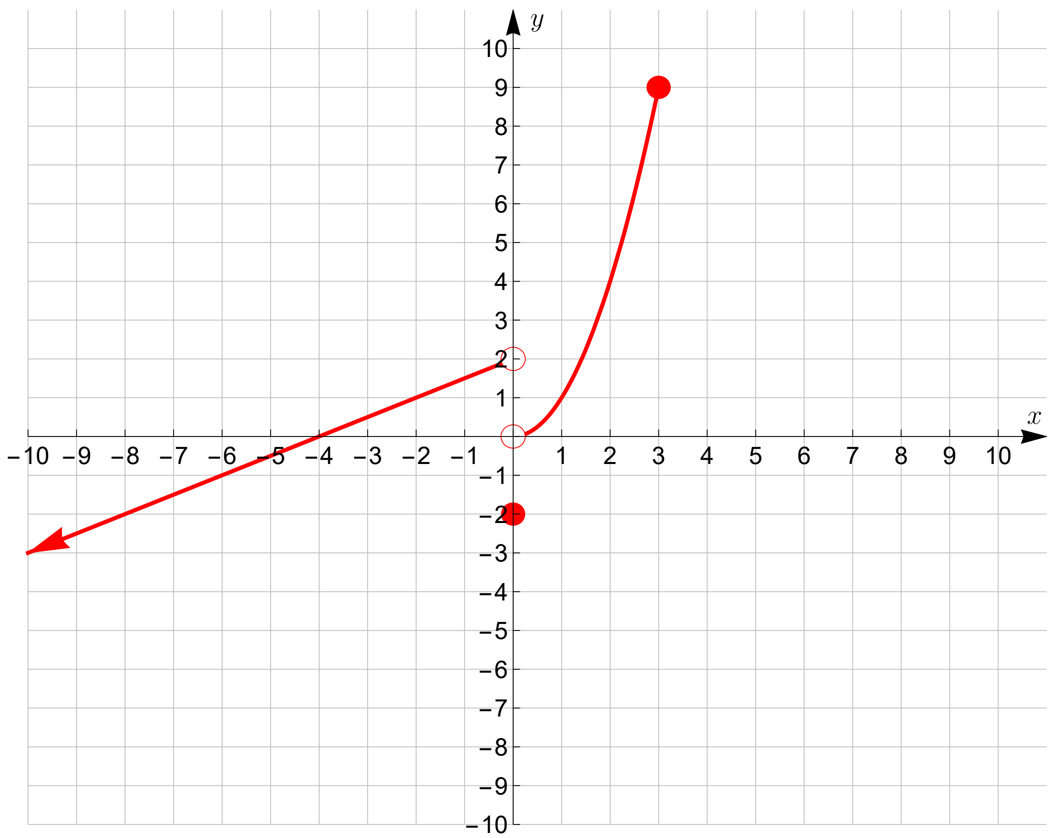


5. Use the following function to answer parts (a) and (b).

$$f(x) = -\sqrt{x+1} - 2$$

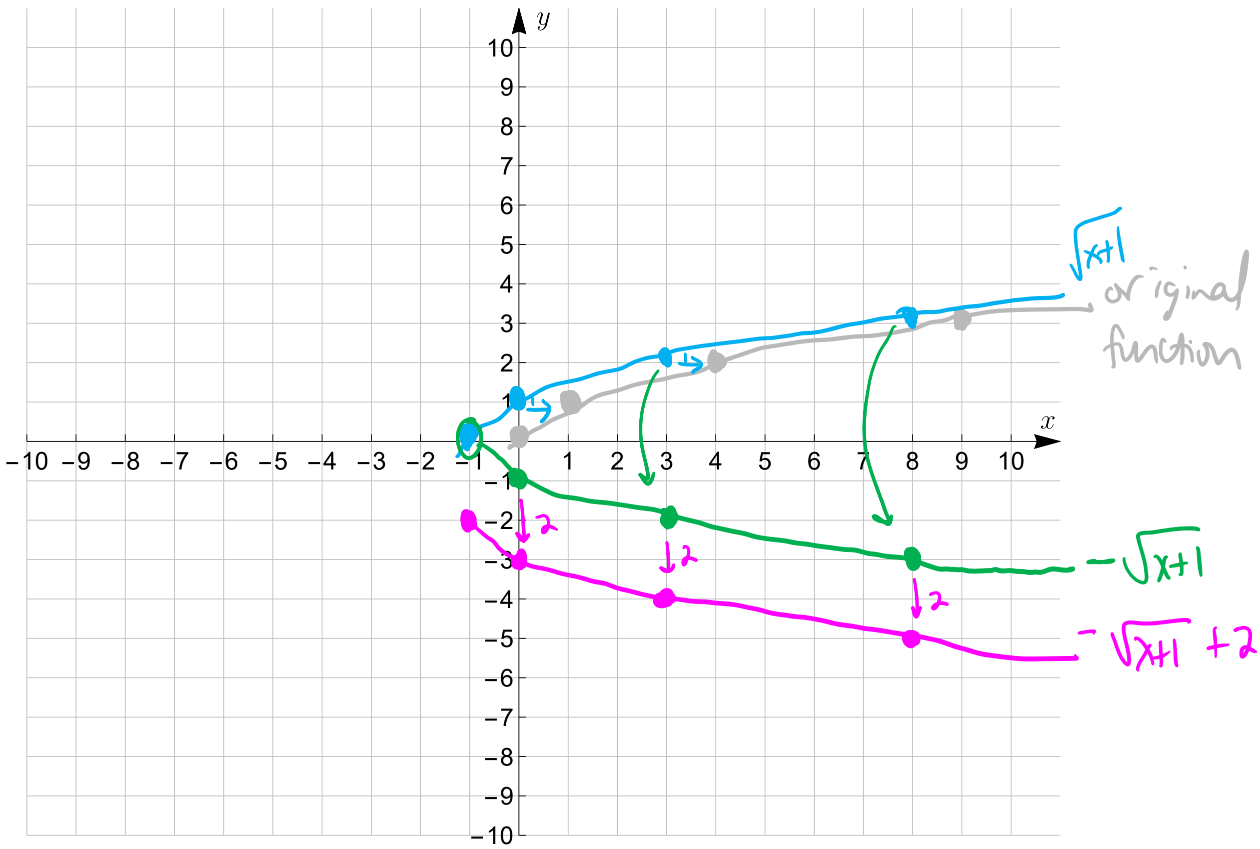
(a) State which transformations have been applied, and in which order they have been applied, to the function $g(x) = \sqrt{x}$ to get the function $f(x)$ given above. **Explain how you know.** (4 points)

1st: Translation left by 1 because of the +1 with the x (horizontal is opposite what you'd expect so plus goes left)

2nd: Vertical reflection because of the negative in front of the entire function

3rd: Translation down by 2 because of the -2 subtracted from the entire function

(b) Graph the function $f(x)$ given above. (6 points)



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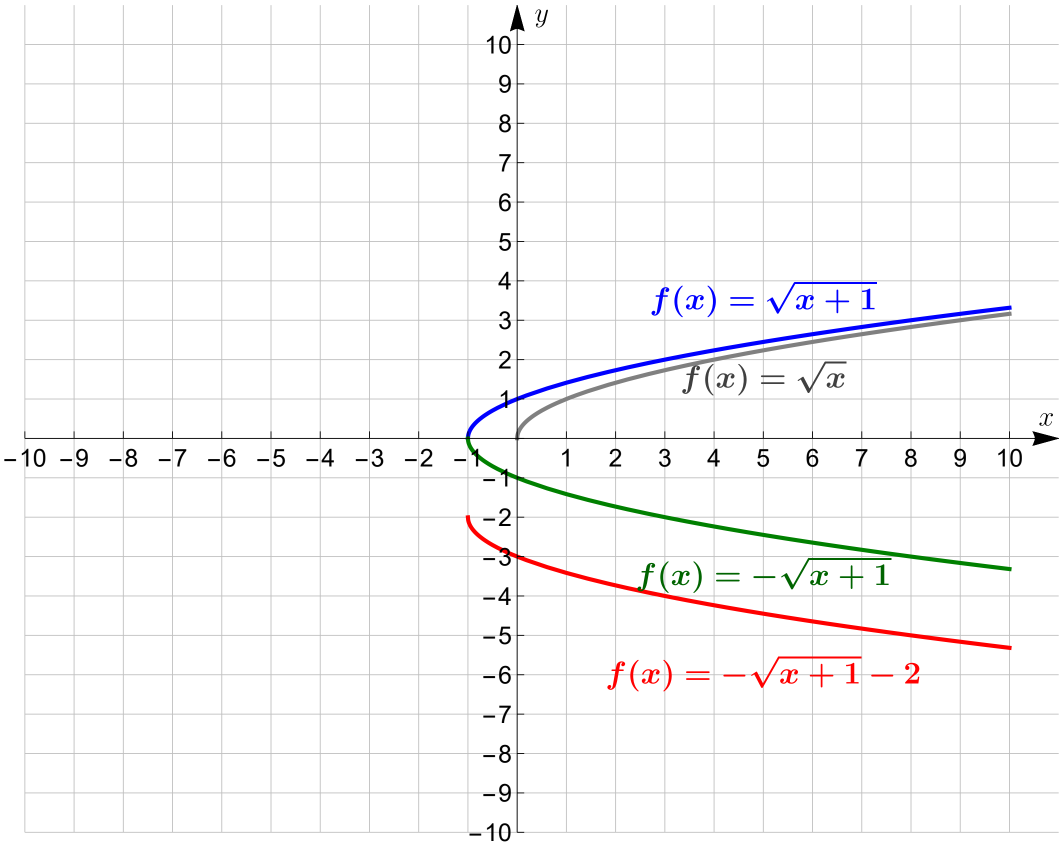


6. Use the following function to answer parts (a) and (b).

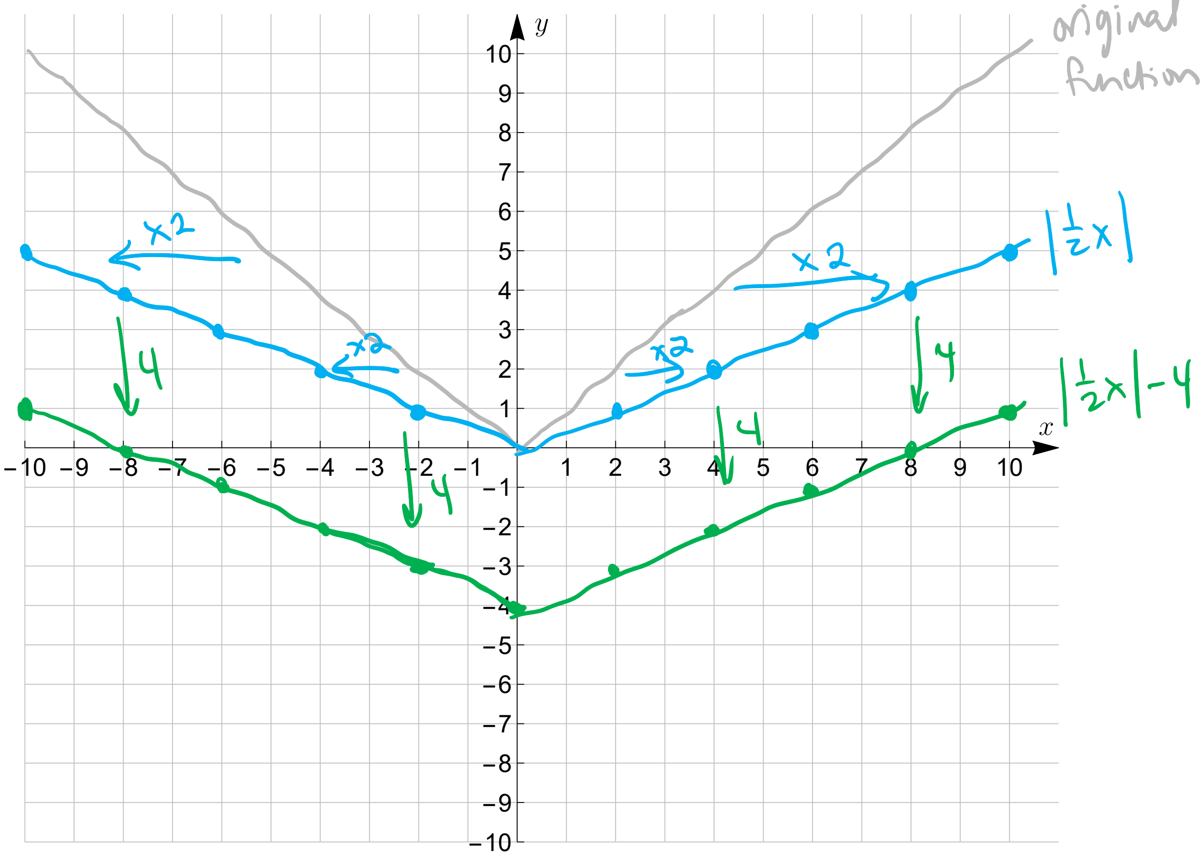
$$f(x) = \left|\frac{1}{2}x\right| - 4$$

(a) State which transformations have been applied, and in which order they have been applied, to the function $g(x) = |x|$ to get the function $f(x)$ given above. **Explain how you know.** (4 points)

1st: Horizontal stretch by 2 because the $\frac{1}{2}$ is multiplied by the x and horizontal is opposite what you'd expect

2nd: Translation down 4 because the -4 is subtracted from the entire function

(b) Graph the function $f(x)$ given above. (6 points)



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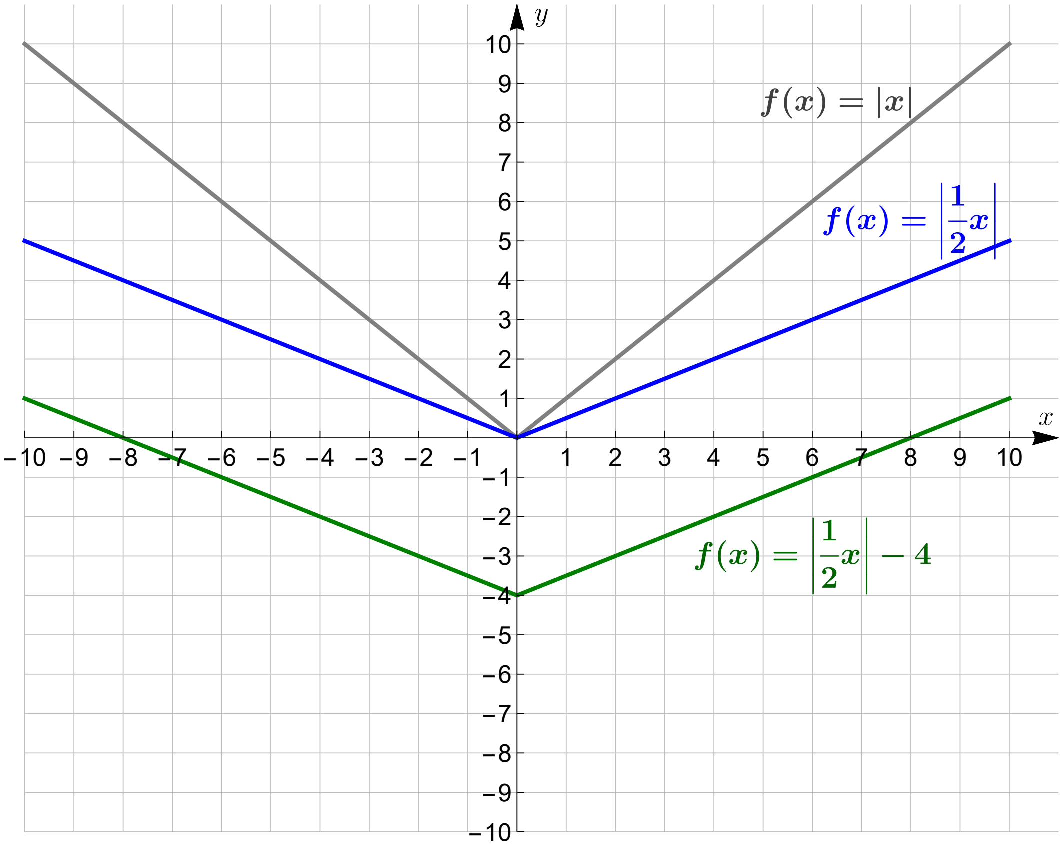


7. Use the following function to answer parts (a) - (d).

$$f(x) = 2x^2 - x + 5$$

(a) Find $f(3)$. (2 points)

$f(3) = 2(3)^2 - 3 + 5 = 2(9) - 3 + 5 = 18 - 3 + 5 = 20$

(b) Find $f(-2)$. (2 points)

$f(-2) = 2(-2)^2 - (-2) + 5 = 2(4) + 2 + 5 = 8 + 2 + 5 = 15$

(c) Find $f(x+h)$. (3 points)

$f(x+h) = 2(x+h)^2 - (x+h) + 5 = 2(x^2 + 2xh + h^2) - x - h + 5$

$= 2x^2 + 4xh + 2h^2 - x - h + 5$

(d) Find the average rate of change of f from $x = -2$ to $x = 3$. (5 points)

$ARC = \dfrac{f(3) - f(-2)}{3 - (-2)} = \dfrac{20 - 15}{3 + 2} = \dfrac{5}{5} = 1$

8. Find the equation of the line, in slope-intercept form, passing through the following points. (8 points)

$$(-1, 10) \text{ and } (3, -2)$$

$$m = \frac{-2-10}{3-(-1)} = \frac{-12}{4} = -3$$

Option #1: $y - y_1 = m(x - x_1)$

$$y - 10 = -3(x-(-1))$$

$$y - 10 = -3(x+1)$$

$$y - 10 = -3x - 3$$

$$+10 \qquad +10$$

$$y = -3x + 7$$

Note
Also could have plugged in the point $(3, -2)$

Option #2: $y = mx + b$

$$y = -3x + b$$

$$10 = -3(-1) + b$$

$$10 = 3 + b$$

$$-3 \quad -3$$

$$7 = b$$

$$y = -3x + 7$$

9. Graph each of the following lines. (6 points each)

(a) $y = -\frac{2}{5}x + 1$

y-int = 1

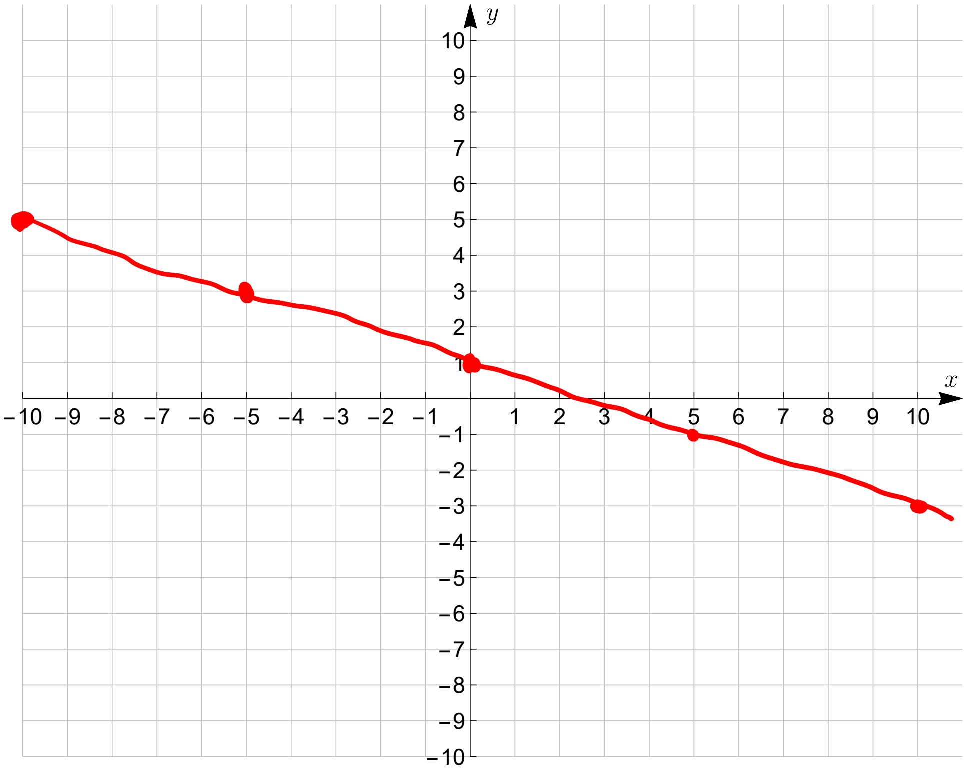
slope = $-\frac{2}{5}$

↓

down 2, right 5

OR

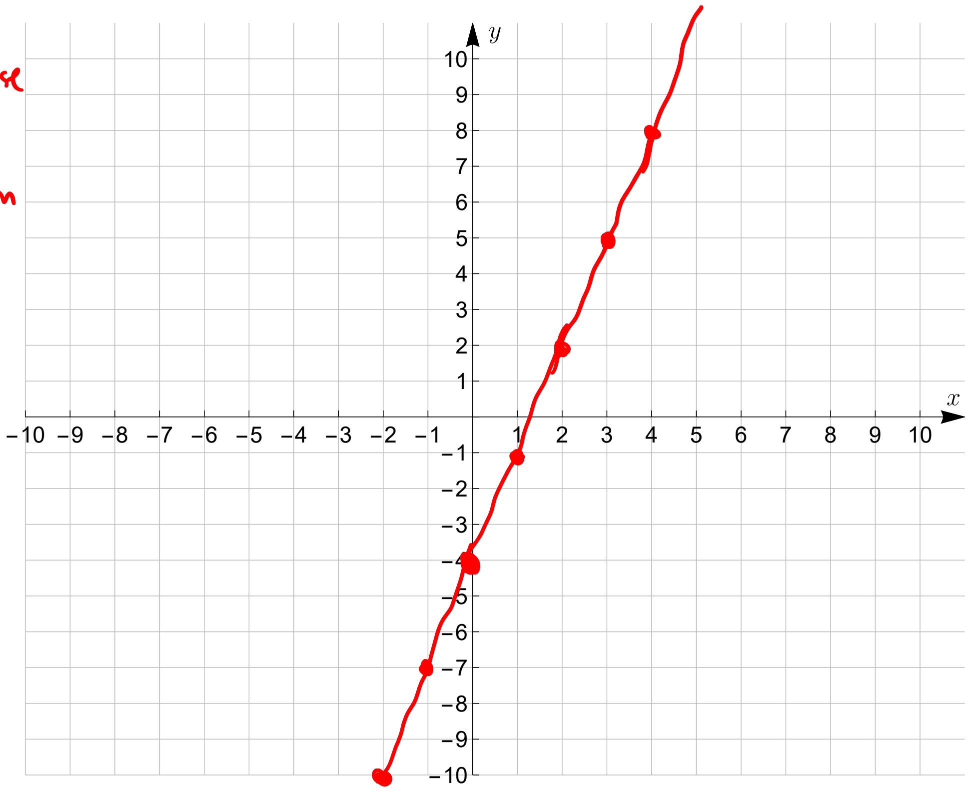
up 2, left 5



(b) $y = 3x - 4$

y-int = 4

slope = 3 = $\frac{3}{1}$ ← rise, ← run



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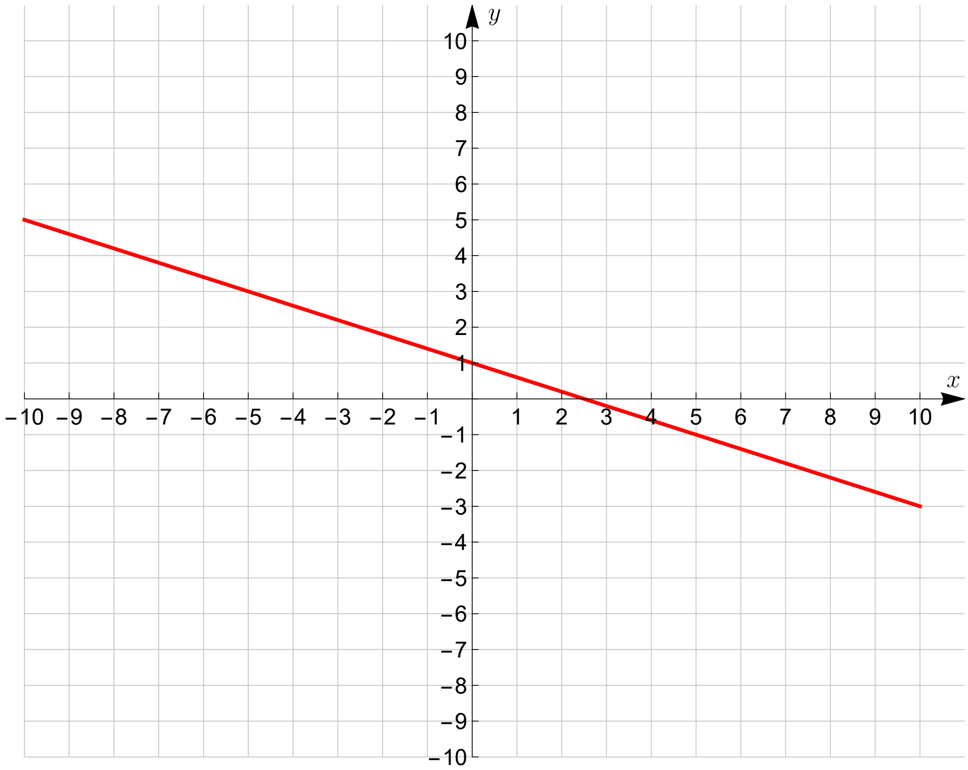
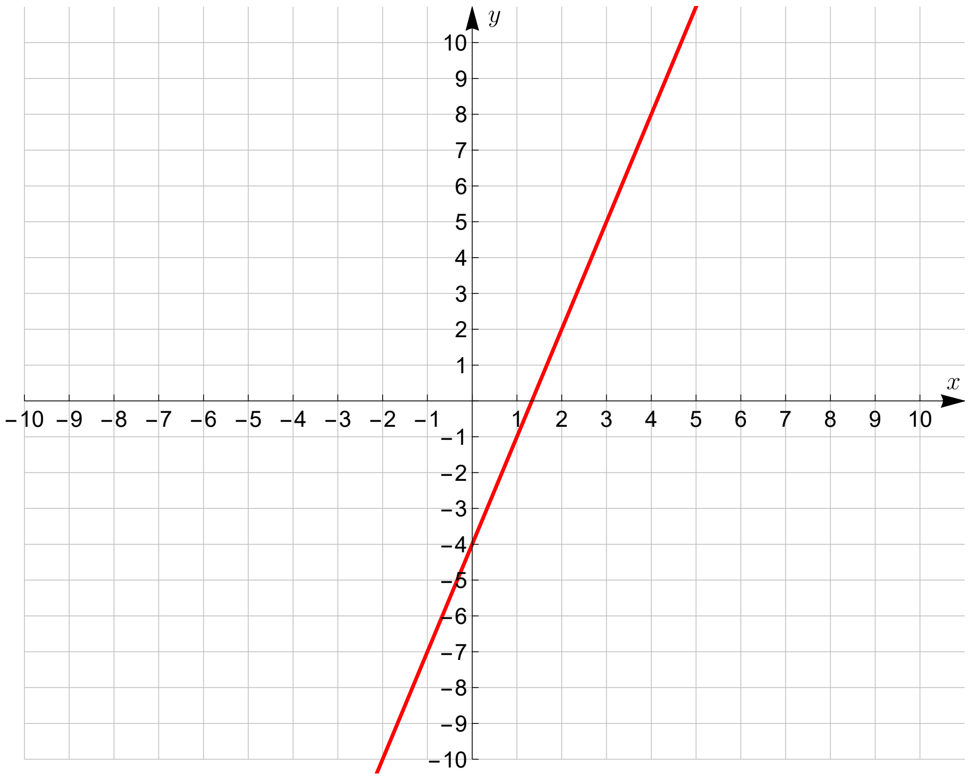
(a) $y = -\frac{2}{5}x + 1$



(b) $y = 3x - 4$



10. Use the following functions to answer parts (a) and (b). (10 points)

$$f(x) = 2x^2 - x + 1 \text{ and } g(x) = -4x + 2$$

(a) Find $(f \circ g)(x)$.

$(f \circ g)(x) = f(g(x)) = f(-4x+2)$

$= 2(-4x+2)^2 - (-4x+2) + 1$

$= 2(16x^2 - 16x + 4) + 4x - 2 + 1$

$= 32x^2 \underline{-32x} \underline{+8} \underline{+4x} \underline{-2} + \underline{1}$

$= 32x^2 - 28x + 7$

(b) Find $(g \circ f)(x)$.

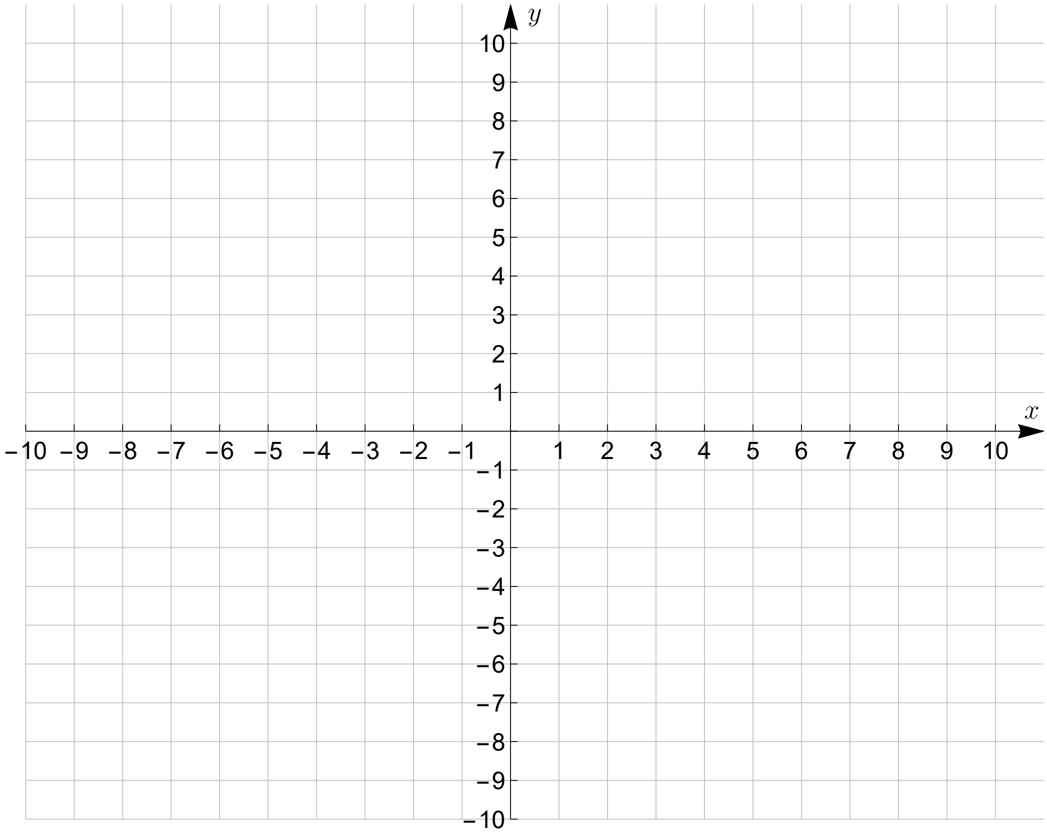
$(g \circ f)(x) = g(f(x)) = g(2x^2 - x + 1)$

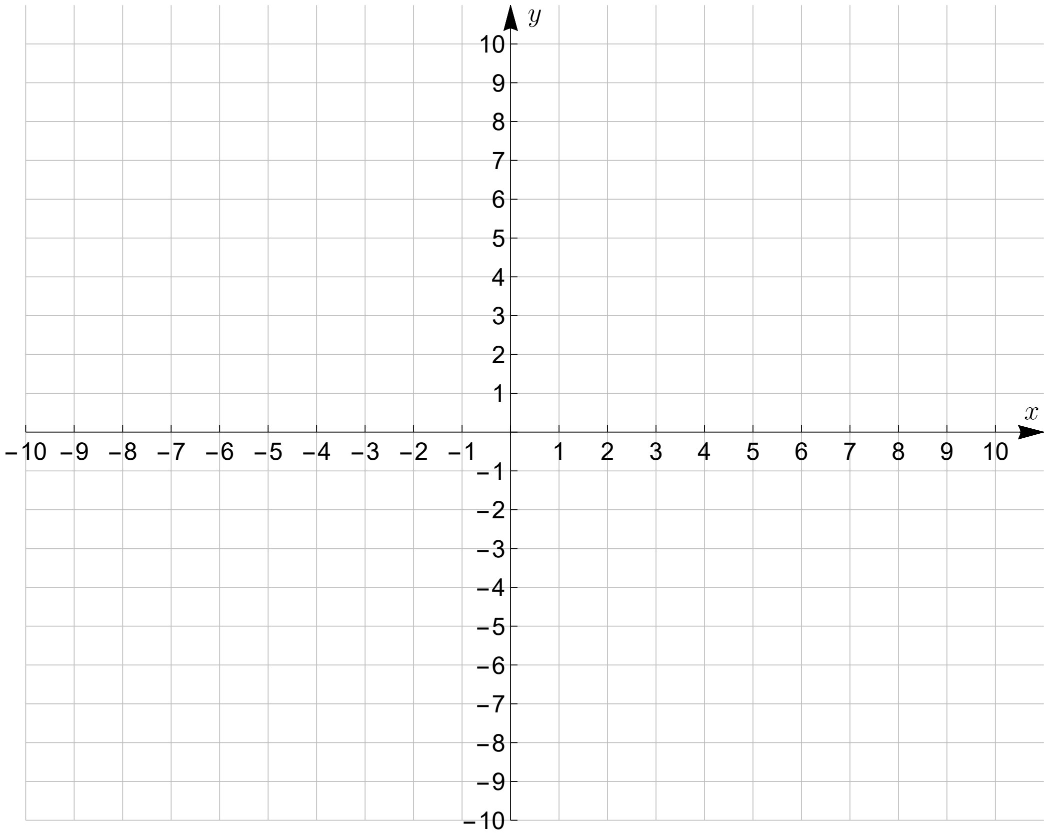
$= -4(2x^2 - x + 1) + 2$

$= -8x^2 + 4x - 4 + 2$

$= -8x^2 + 4x - 2$

Extra Blank Graphs.





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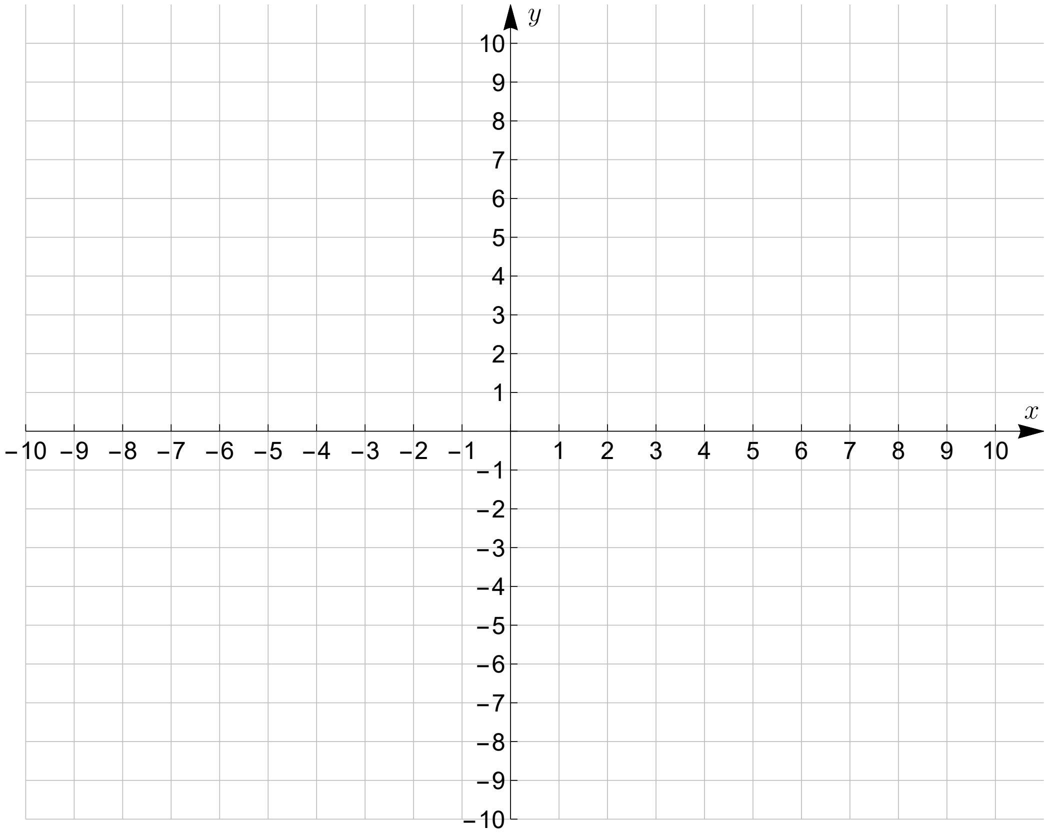

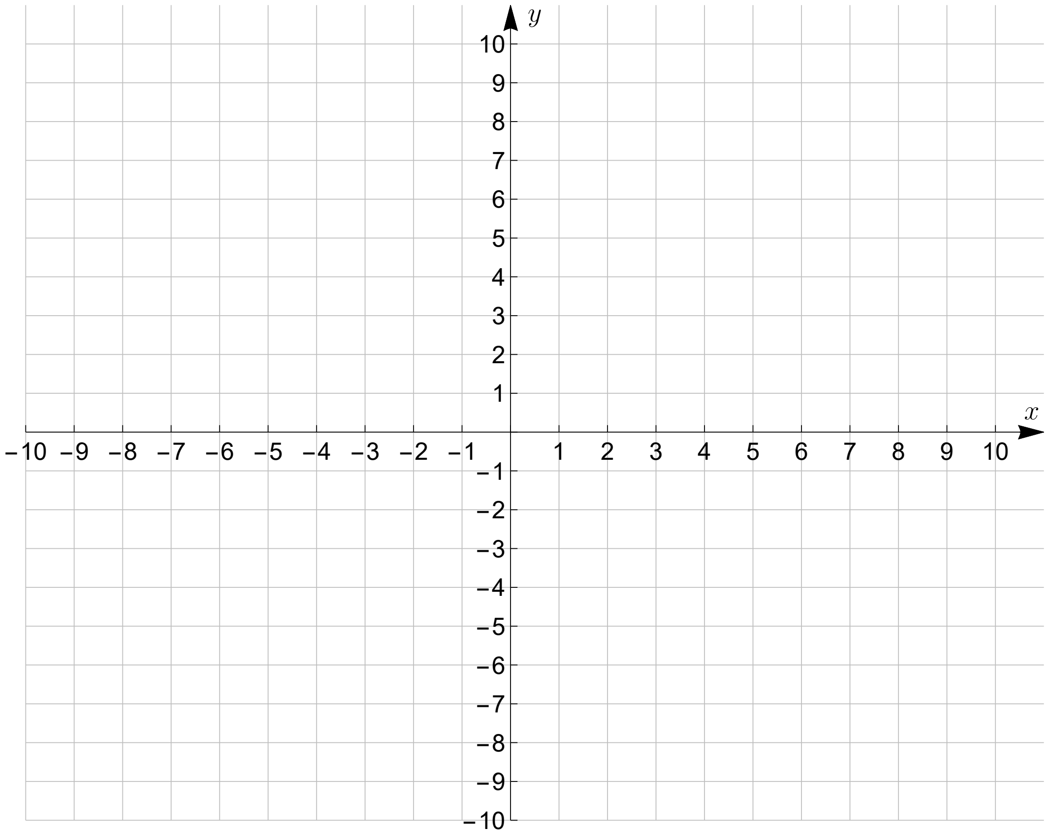

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