

Math 126 - Spring 2020 - Exam #2 (Take-Home)
DUE WEDNESDAY, MARCH 25, 2020 BY 11:59PM

Name: Answer Key

ID# _____

HONOR CODE: On my honor, I have neither given nor received any aid on this examination that is not explicitly allowed in the instructions.

Signature: _____

Instructions: You may review the lecture videos and use your notes, the textbook, and ALEKS homework when working on this exam. You may **not** receive help from anyone else. You may submit your answers and scratch work either on a printed copy of this exam or on your own paper. If you use your own paper, you do **not** need to copy the question; just be sure you clearly label which question the scratch work and answer belong to. If I can't tell with certainty which question any scratch work or answer belongs to, you will not receive credit for that work or answer. To submit your scratch work and answers, you can either scan your work (if you have access to a scanner) or take pictures with your cell phone, then email me your scans or pictures. **BE SURE THE WRITING IN YOUR SCANS OR PICTURES IS DARK ENOUGH THAT I CAN EASILY READ WHAT YOU'VE WRITTEN. IF I CAN'T READ WHAT YOU'VE WRITTEN, I CAN'T CREDIT FOR IT.** Make sure your final answers are clearly labeled. **SHOW ALL WORK ON THIS EXAM IN ORDER TO RECEIVE FULL CREDIT!!!**

No.	Score
1	/14
2	/8
3	/14
4	/10
5	/8
6	/10
7	/18
8	/18
Bonus	/10
Total	/100

1. Given

$$f(x) = 3(x - 2)^2 - 3$$

- (a) State whether the graph opens upward or downward. **Explain your answer.** (2 points)

upward because the leading coefficient is 3, which is positive

- (b) State the vertex. (2 points)

(2, -3)

- (c) State whether the function has a maximum or minimum value, where that maximum or minimum value occurs, and what the maximum or minimum value is. (3 points)

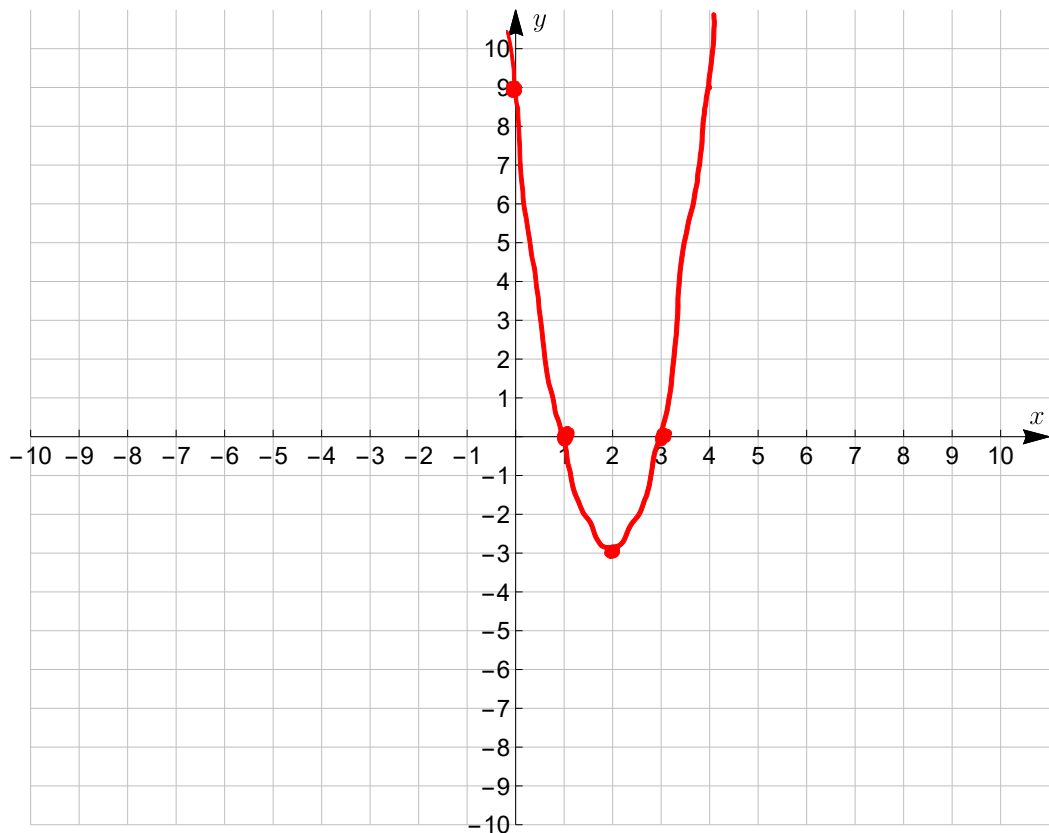
Since the function opens upward it has a minimum value of -3 at $x = 2$

- (d) State the domain and range of the function. (3 points)

Domain = $(-\infty, \infty)$

Range = $[-3, \infty)$

(e) Graph the function. (4 points)



$$\text{x-intercepts: } 0 = \frac{3(x-2)^2 - 3}{+3} \quad \frac{-3}{+3}$$

$$\frac{3}{-3} = \frac{3(x-2)^2}{3}$$

$$\pm \sqrt{1} = \sqrt{(x-2)^2}$$

$$\frac{\pm 1}{+2} = \frac{x-2}{+2}$$

$$x = 2 \pm 1 \Rightarrow x = 2+1 \text{ and } x = 2-1$$

$$x = 3 \text{ and } x = 1$$

$$\text{y-int: } f(0) = 3(0-2)^2 - 3$$

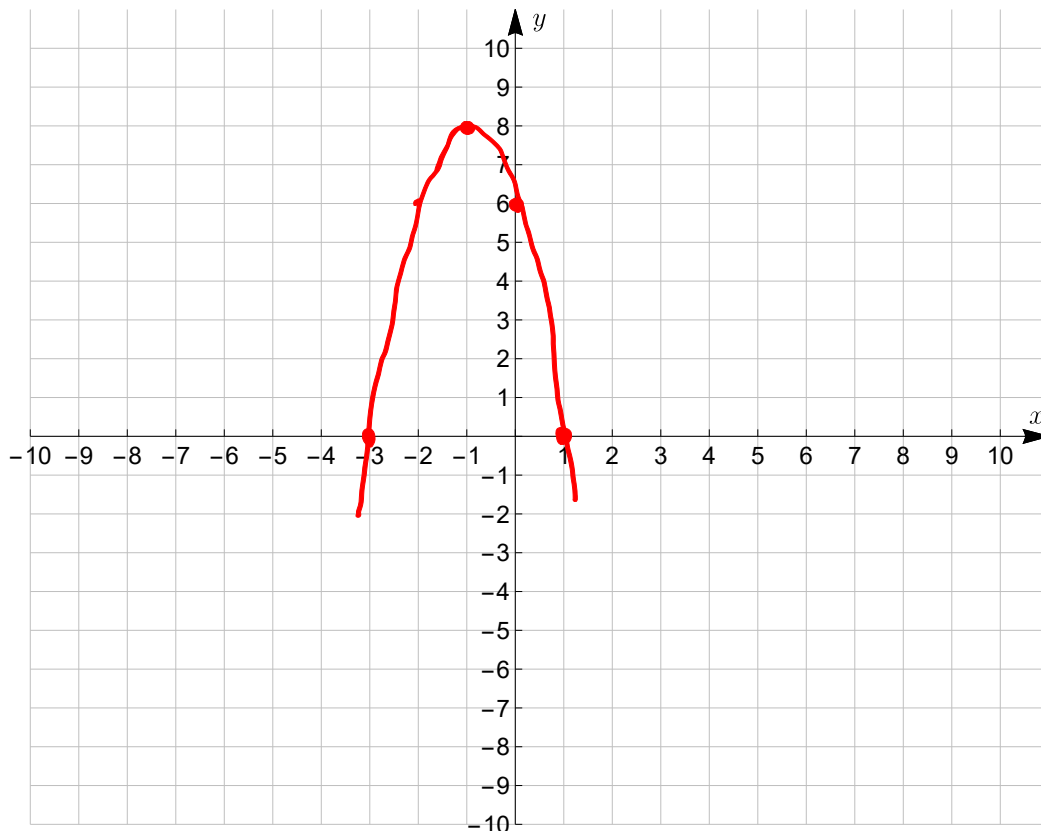
$$= 3(4) - 3$$

$$= 12 - 3$$

$$= 9$$

2. Write the following quadratic function in vertex form then graph the function. (8 points)

$$f(x) = -2x^2 - 4x + 6$$



$$\text{Vertex: } x = -\frac{b}{2a} = -\frac{-4}{2(-2)} = -1$$

$$\begin{aligned} f(-1) &= -2(-1)^2 - 4(-1) + 6 \\ &= -2 + 4 + 6 \\ &= 8 \end{aligned}$$

$$\text{Vertex} = (-1, 8)$$

$$f(x) = -2(x - (-1))^2 + 8$$

$$f(x) = -2(x+1)^2 + 8$$

$$\text{x-ints: } 0 = -2(x+1)^2 + 8$$

$$\begin{aligned} -8 &= -2(x+1)^2 \\ -2 &= \frac{-2(x+1)^2}{-2} \end{aligned}$$

$$\pm \sqrt{4} = \sqrt{(x+1)^2}$$

$$\pm 2 = \frac{x+1}{-1}$$

$$x = -1 \pm 2$$

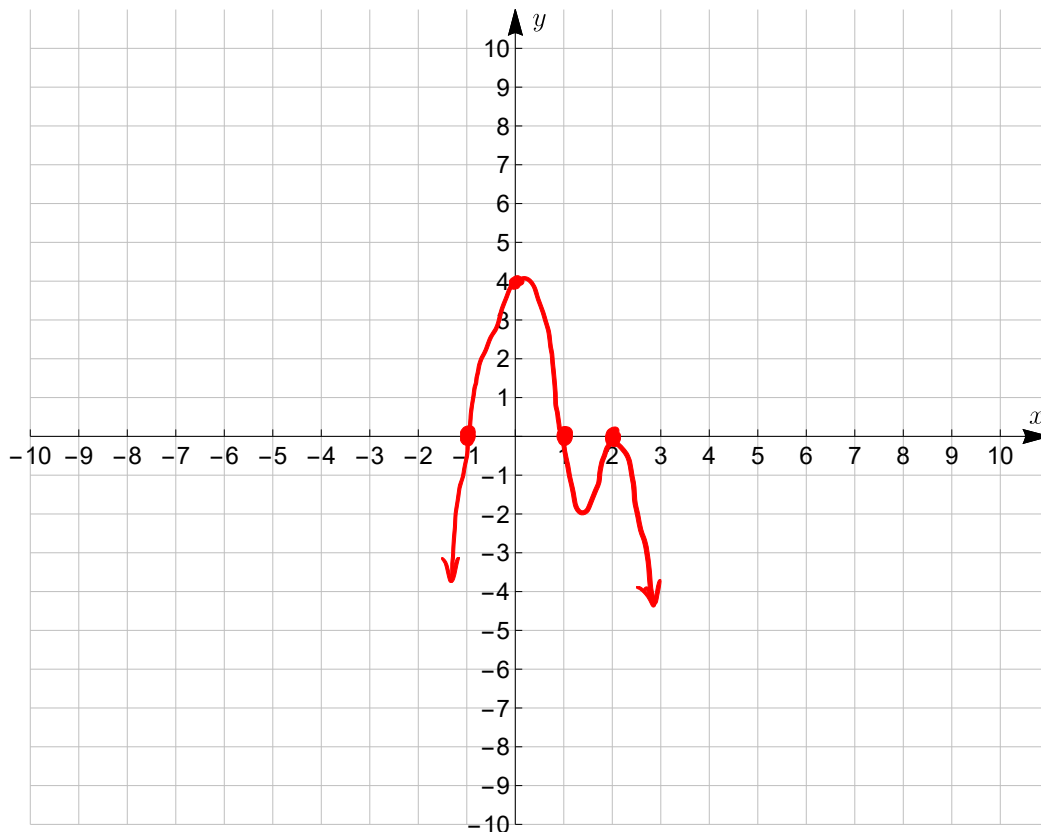
$$x = -1 + 2 \text{ and } x = -1 - 2$$

$$x = 1 \text{ and } x = -3$$

$$\begin{aligned} \text{y-int: } f(0) &= -2(0)^2 - 4(0) + 6 \\ &= 6 \end{aligned}$$

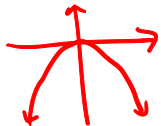
3. Graph the following function. (14 points)

$$f(x) = -(x-1)(x+1)^3(x-2)^2$$



end behavior: $f(x) = -x \cdot x^3 \cdot x^2 = -x^6$ even
 negative

think of $f(x) = -x^2$



f is falling as $x \rightarrow -\infty$ and as $x \rightarrow \infty$

Zeros: $0 = -(x-1)(x+1)^3(x-2)^2$

$x=1$ mult=1 crosses
 $x=-1$ mult=3 crosses
 $x=2$ mult=2 touches and turns around

y-int: $f(0) = -(-1)(1)^3(0-2)^2$
 $= -(-1)(1)(4) = 4$

4. Find the quotient of each of the following. (5 points each)

(a) $(2x^4 - 6x^3 + 2x + 8) \div (x^2 - 2)$

$$\begin{array}{r}
 2x^2 - 6x + 4 \\
 x^2 + 0x - 2 \overline{) 2x^4 - 6x^3 + 0x^2 + 2x + 8} \\
 \underline{- 2x^4 + 0x^3 + 4x^2} \quad \downarrow \\
 -6x^3 + 4x^2 + 2x + 8 \\
 \underline{+ 6x^3 + 0x^2 + 12x} \quad \downarrow \\
 4x^2 - 10x + 8 \\
 \underline{- 4x^2 + 0x + 8} \\
 -10x + 16
 \end{array}$$

$$2x^2 - 6x + 4 + \frac{-10x + 16}{x^2 - 2}$$

(b) $(3x^5 - 4x^4 + 2x^3 - x^2 + x + 4) \div (x + 1)$

$$\begin{array}{r}
 -1 \overline{) 3 \quad -4 \quad 2 \quad -1 \quad 1 \quad 4} \\
 \underline{\downarrow -3 \quad 7 \quad -9 \quad 10 \quad -11} \\
 3 \quad -7 \quad 9 \quad -10 \quad 11 \quad -7
 \end{array}$$

$$3x^4 - 7x^3 + 9x^2 - 10x + 11 - \frac{7}{x+1}$$

5. Determine the number of positive zeros and the number negative zeros the following polynomial function can have, then list the potential rational zeros of the function. (8 points)

$$f(x) = -2x^3 + 3x^2 + 2x + 12$$

$$f(x) = -2x^3 + 3x^2 + 2x + 12$$

1 sign change \Rightarrow 1 positive zero

$$f(-x) = -2(-x)^3 + 3(-x)^2 + 2(-x) + 12$$

$$= 2x^3 + 3x^2 - 2x + 12$$

2 sign changes \Rightarrow 2 or 0 negative zeros

$$\text{Factors of } 12 = \pm(1, 2, 3, 4, 6, 12)$$

$$\text{Factors of } 2 = \pm 1, \pm 2$$

$$\text{Possibilities} = \frac{\text{Factors of } 12}{\text{Factors of } 2} = \frac{\pm(1, 2, 3, 4, 6, 12)}{\pm 1, \pm 2}$$

$$= \pm\left(\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{6}{1}, \frac{12}{1}, \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{6}{2}, \frac{12}{2}\right)$$

$$= \pm\left(1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}\right)$$

6. Find all the roots of the following polynomial function and write the function in factored form. (10 points)

$$f(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$$

$$f(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$$

2 sign changes \Rightarrow 2 or 0 positive zeros

$$f(-x) = (-x^4) - 4(-x)^3 - 3(-x)^2 + 10(-x) + 8$$

$$= x^4 + 4x^3 - 3x^2 - 10x + 8$$

2 sign changes \Rightarrow 2 or 0 positive zeros

Factors of 8 = $\pm(1, 2, 4, 8)$

Factors of 1 = ± 1

$$\text{Possibilities} = \frac{\text{Factors of 8}}{\text{Factors of 1}} = \frac{\pm(1, 2, 4, 8)}{\pm 1} = \pm\left(\frac{1}{1}, \frac{2}{1}, \frac{4}{1}, \frac{8}{1}\right) = \pm(1, 2, 4, 8)$$

$$\begin{array}{r} 1 \mid 1 \quad -4 \quad -3 \quad 10 \quad 8 \\ \quad \downarrow \quad 1 \quad -3 \quad -6 \quad 4 \\ \hline 1 \quad -3 \quad -6 \quad 4 \quad \cancel{8} \end{array}$$

$$\begin{array}{r} 2 \mid 1 \quad -4 \quad -3 \quad 10 \quad 8 \\ \quad \downarrow \quad 2 \quad -4 \quad -14 \quad -8 \\ \hline 1 \quad -2 \quad -7 \quad -4 \quad \boxed{0} \end{array}$$

$\hookrightarrow x=2$ is a zero
 $x-2$ is a factor

$$x^4 - 4x^3 - 3x^2 + 10x + 8 = (x-2)(x^3 - 2x^2 - 7x - 4)$$

$$\begin{array}{r} 2 \mid 1 \quad -2 \quad -7 \quad -4 \\ \quad \downarrow \quad 2 \quad 0 \quad -14 \\ \hline 1 \quad 0 \quad -7 \quad \cancel{-4} \end{array}$$

$$\begin{array}{r} 4 \mid 1 \quad -2 \quad -7 \quad -4 \\ \quad \downarrow \quad 4 \quad 8 \quad 4 \\ \hline 1 \quad 2 \quad 1 \quad \boxed{0} \end{array}$$

$\hookrightarrow x=4$ is a zero
 $x-4$ is a factor

$$x^4 - 4x^3 - 3x^2 + 10x + 8 = (x-2)(x-4)\underbrace{(x^2 + 2x + 1)}_{(x+1)(x+1)}$$

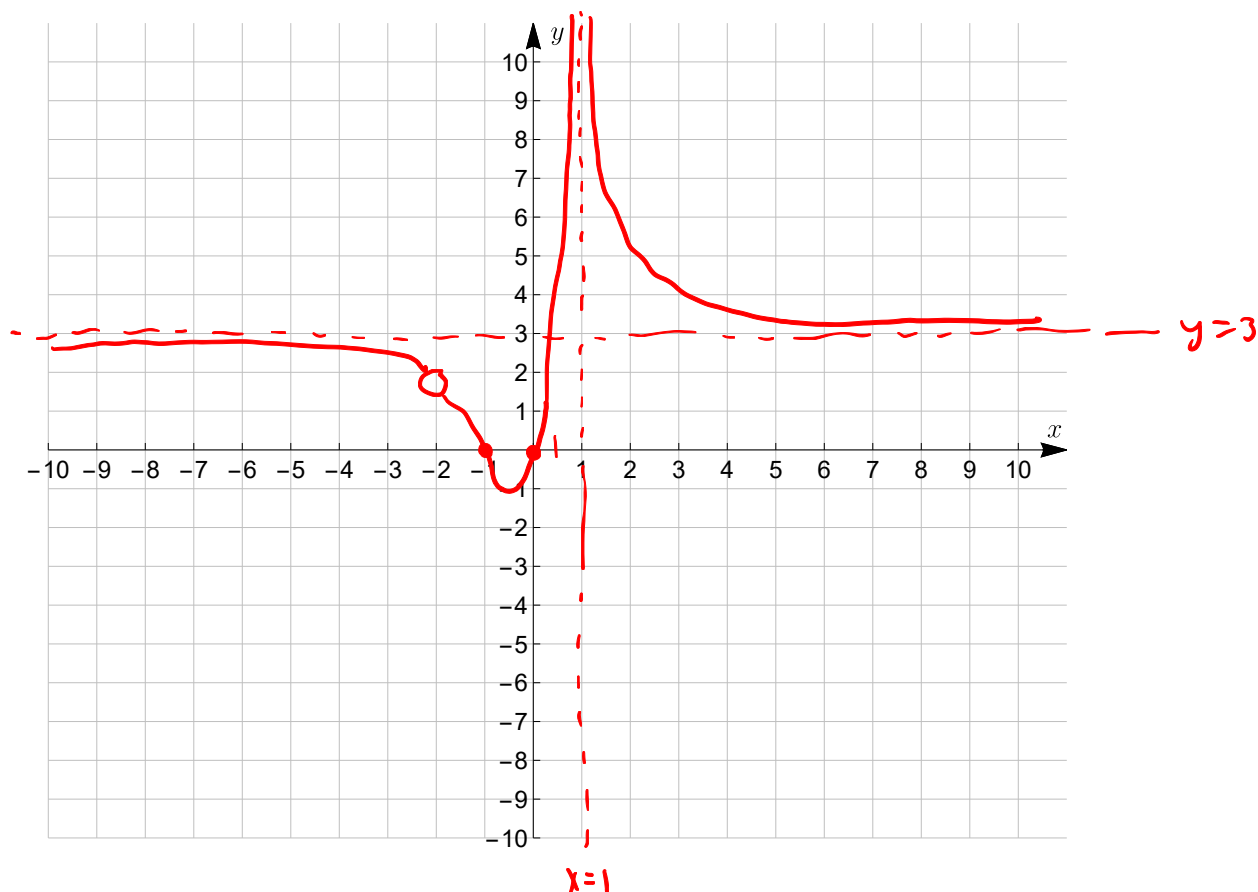
$$x^4 - 4x^3 - 3x^2 + 10x + 8 = (x-2)(x-4)\underbrace{(x+1)^2}_{(x+1)^2 = 0 \Rightarrow x = -1}$$

zeros: $x=2, x=4, x=-1$

6. (continued)

7. Graph the following function. (18 points)

$$f(x) = \frac{3x^3 + 9x^2 + 6x}{x^3 - 3x + 2} = \frac{3x(x+1)(x+2)}{(x+2)(x-1)^2}$$



Degree of numerator = 3 } same degree \Rightarrow horizontal asymptote: $y = \frac{3}{1} = 3$
 Degree of denominator = 3

$$(x+2)(x-1)^2 = 0$$

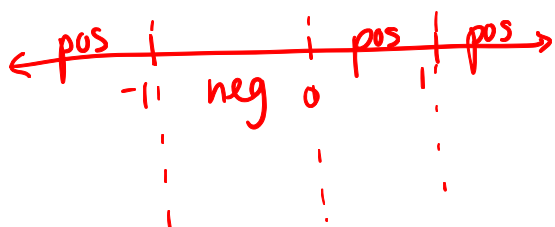
$x = -2$ } $x = 1$
 Cancels with $x-2$ in numerator } doesn't cancel
 \downarrow } \downarrow
 hole in graph } vertical asymptote

$$f(x) = \frac{3x(x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-1)^2} = \frac{3x(x+1)}{(x-1)^2}$$

x-int: $0 = 3x(x+1)$
 \downarrow } \downarrow
 $x = 0$ } $x = -1$

7. (continued)

$$x = -2, \quad x = -0.5, \quad x = 0.5, \quad x = 2$$



$$f(-2) = \frac{(\text{neg})(\text{neg})}{\text{pos}} = \frac{\text{pos}}{\text{pos}} = \text{pos}$$

$$f(-0.5) = \frac{(\text{neg})(\text{pos})}{\text{pos}} = \frac{\text{neg}}{\text{pos}} = \text{neg}$$

$$f(0.5) = \frac{(\text{pos})(\text{pos})}{\text{pos}} = \frac{\text{pos}}{\text{pos}} = \text{pos}$$

$$f(2) = \frac{(\text{pos})(\text{pos})}{\text{pos}} = \frac{\text{pos}}{\text{pos}} = \text{pos}$$

8. Solve each of the following inequalities. (6 points each)

(a) $3x^2 - 4 \geq 4x$

$-4x \quad -4x$

$3x^2 - 4x - 4 \geq 0$

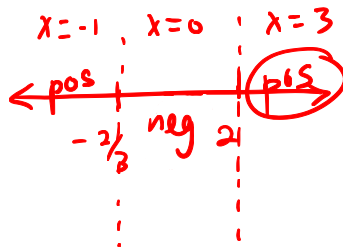
$(3x+2)(x-2) \geq 0$ → looking for positive or zero

$3x+2=0$

$x = -\frac{2}{3}$

$x-2=0$

$x=2$



$(-\infty, -\frac{2}{3}] \cup [2, \infty)$

$f(-1) = (\text{neg})(\text{neg}) = \text{pos}$

$f(0) = (\text{pos})(\text{neg}) = \text{neg}$

$f(3) = (\text{pos})(\text{pos}) = \text{pos}$

(b) $(x-4)^2(x-6)(x+1) < 0$ → looking for negative

$x=4$

$x=6$

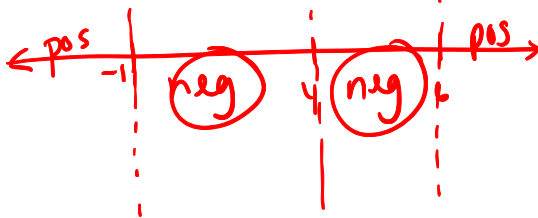
$x=-1$

$x=-2$

$x=0$

$x=5$

$x=7$



$f(-2) = (\text{pos})(\text{neg})(\text{neg}) = \text{pos}$

$f(0) = (\text{pos})(\text{neg})(\text{pos}) = \text{neg}$

$f(5) = (\text{pos})(\text{neg})(\text{pos}) = \text{neg}$

$f(7) = (\text{pos})(\text{pos})(\text{pos}) = \text{pos}$

$(-1, 4) \cup (4, 6)$

(c) $\frac{(x+3)^2}{(x-2)(x+4)} \leq 0$ → looking for negative or zero

$(x+3)^2 = 0$

$x = -3$

v.g. so not included

$(x-2)(x+4) = 0$

$x = 2, x = -4$

zero so included

$x = -5, x = -3.5, x = 0, x = 3$



$f(-5) = \frac{\text{pos}}{(\text{neg})(\text{neg})} = \frac{\text{pos}}{\text{pos}} = \text{pos}$

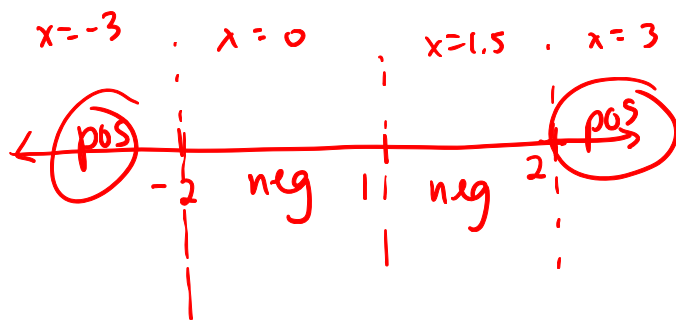
$f(-3.5) = \frac{\text{pos}}{(\text{neg})(\text{pos})} = \frac{\text{pos}}{\text{neg}} = \text{neg}$

$f(0) = \frac{\text{pos}}{(\text{neg})(\text{pos})} = \frac{\text{pos}}{\text{neg}} = \text{neg}$

$f(3) = \frac{\text{pos}}{(\text{pos})(\text{pos})} = \frac{\text{pos}}{\text{pos}} = \text{pos}$

$(-4, -3] \cup [-3, 2) = (-4, 2)$

Bonus. (continued)



$$g(-3) = (\text{pos})(\text{neg})(\text{neg}) = \text{pos}$$

$$g(0) = (\text{pos})(\text{neg})(\text{pos}) = \text{neg}$$

$$g(1.5) = (\text{pos})(\text{neg})(\text{pos}) = \text{neg}$$

$$g(3) = (\text{pos})(\text{pos})(\text{pos}) = \text{pos}$$

$$\text{Domain} = (-\infty, -2] \cup [2, \infty)$$