Math 126 - Spring 2020 - Exam #3 (Take-Home)
DUE WEDNESDAY, MAY 6, 2020 BY 11:59PM

Name: Answer Key

HONOR CODE: On my honor, I have neither given nor received any aid on this examination that is not explicitly allowed in the instructions.

Signature: _____

Instructions: You may review the lecture videos and use your notes, the lecture notes, the textbook, the ALEKS homework, and your calculator when working on this exam. **You may <u>not</u> receive help from anyone else, give help to anyone else, discuss any aspect of the exam or any items related to the exam with anyone else, or use any resources not specified in the previous sentence.** You may submit your answers and scratch work either on a printed copy of this exam or on your own paper. If you use your own paper, you do **not** need to copy the question; just be sure you clearly label which question the scratch work and answer belong to. If I can't tell with certainty which question any scratch work or answer belongs to, you will not receive credit for that work or answer. If you use your own paper, you **DO** need to copy the honor code above and sign it. To submit your scratch work and answers, you can either scan your work (if you have access to a scanner) or take pictures with your cell phone, then email me your scans or pictures. Be sure the writing in your scans or pictures is dark enough and clear enough that I can easily read what you've written. If I can't read what you've written, I can't give you credit for it. Make sure your final answers are clearly labeled. **SHOW ALL WORK ON THIS EXAM IN ORDER TO RECEIVE FULL CREDIT!!!**

No.	Score
1	/12
2	/8
3	/8
4	/8
5	/8
6	/12
7	/22
8	/10
9	/12
Bonus	/10
Total	/100

1. Find the inverse of f. State the domain and range of both f and f^{-1}. (12 points)

$$f(x) = \frac{6x}{x-4}$$

$y = \frac{6x}{x-4}$

$(y-4)x = \frac{6y}{y-4} \cdot (y-4)$

$x(y-4) = 6y$

$xy - 4x = 6y$

$-xy \qquad -xy$

$-4x = 6y - xy$

$\frac{-4x}{6-x} = \frac{y(6-x)}{6-x}$

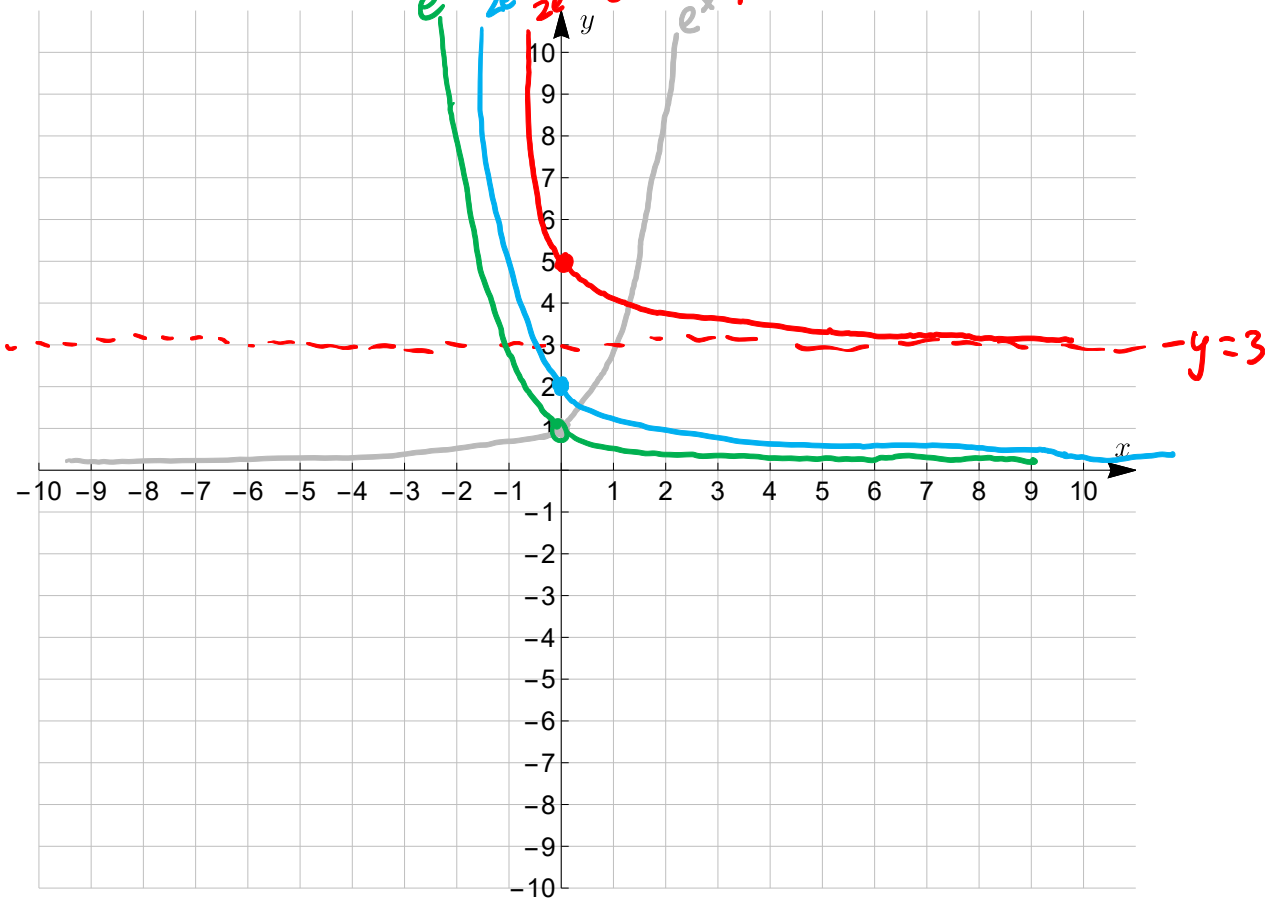
$y = -\frac{4x}{6-x} = \frac{4x}{x-6}$

Domain of $f = (-\infty, 4) \cup (4, \infty) =$ Range of f^{-1}

Domain of $f^{-1} = (-\infty, 6) \cup (6, \infty) =$ Range of f

2. Graph the following function. (8 points)

$$f(x) = 2e^{-x} + 3$$

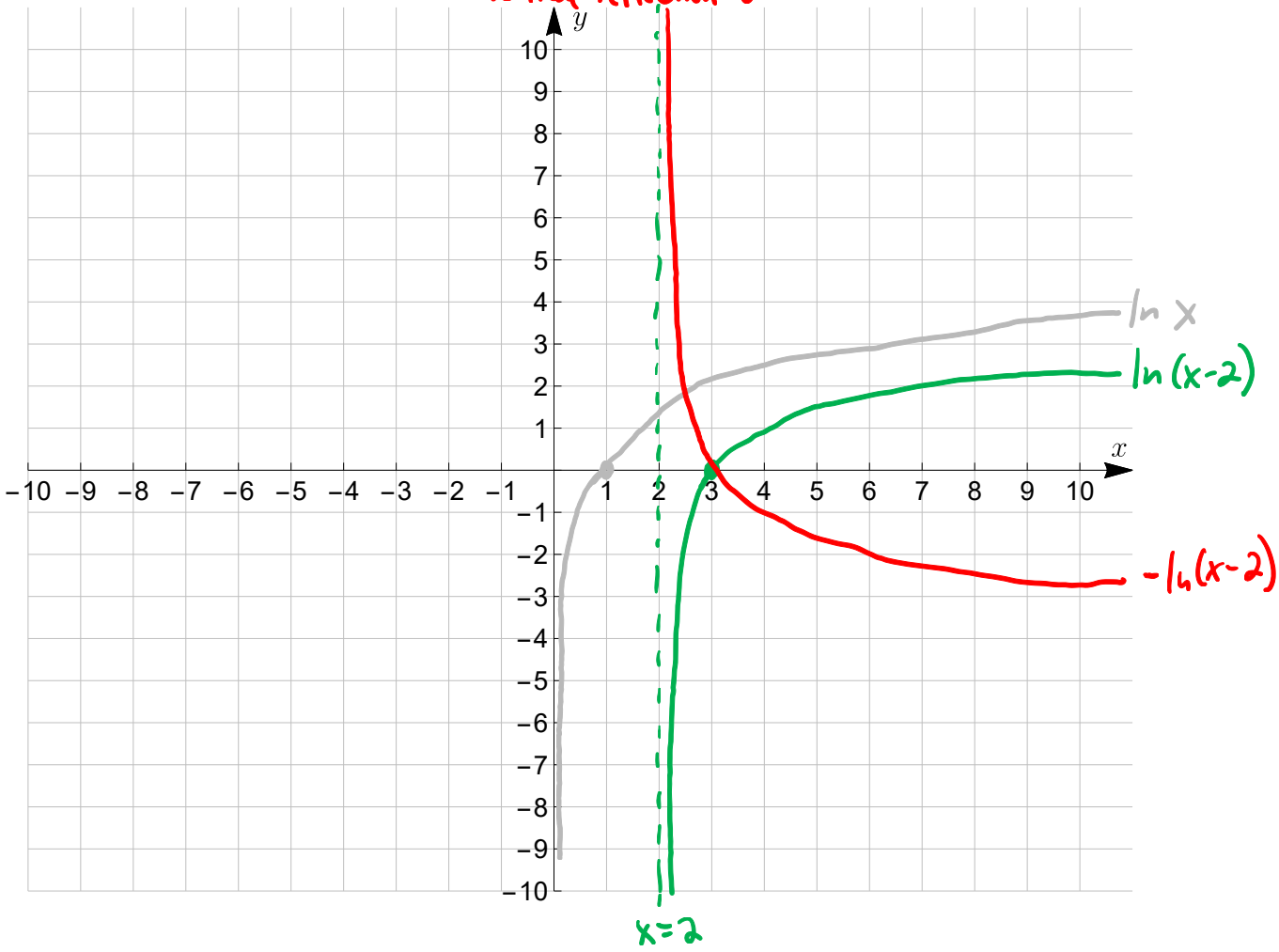


3. Graph the following function. (8 points)

shift right by 2 1st

$$f(x) = -\ln(x-2)$$

vertical reflection 2nd



4. Change the exponential expression to an equivalent expression involving a logarithm. Be sure to use the standard notation for special logarithms. (2 points each)

(a) $3^x = 4.6$

$\log_3 4.6 = x$

(b) $e^{2.2} = M$

$\log_e M = 2.2$

$\ln M = 2.2$

Change the logarithmic expression to an equivalent expression involving an exponent. (3 points each)

(c) $\log_y 6 = 7$

$y^7 = 6$

(d) $\log x = 4$

$\log_{10} x = 4$

$10^4 = x$

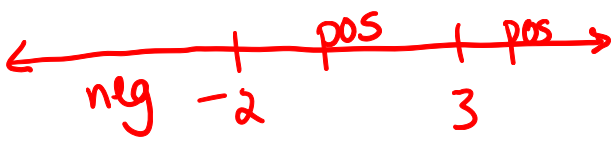
5. Find the domain of the function. (8 points)

$$\log_3\left((x-3)^2(x+2)\right)$$

looking for positive

$(x-3)^2(x+2) > 0$

$x = 3$ $\quad$ $x = -2$



$x = -3$: (pos)(neg) = neg

$x = 0$ (pos)(pos) = pos

$x = 4$ (pos)(pos) = pos

Domain = $(-2, 3) \cup (3, \infty)$

6. (a) Write as a sum/difference of logarithms. Express exponents as factors. Simplify as much as possible. (6 points)

$$\log_4\left(\frac{\sqrt[3]{x^2+1}}{x^2-1}\right)$$

$\log_4(\sqrt[3]{x^2+1}) - \log_4(x^2-1)$

$\log_4((x^2+1)^{1/3}) - \log_4((x+1)(x-1))$

$\frac{1}{3}\log_4(x^2+1) - \log_4(x+1) - \log_4(x-1)$

(b) Write as a single logarithm. Simplify as much as possible. (6 points)

$$\ln\left(x^2+3x+2\right) - 2\ln(x+1) - \ln(x+2)$$

$\ln(x^2+3x+2) - \ln(x+1)^2 - \ln(x+2)$

$\ln\left(\frac{x^2+3x+2}{(x+1)^2(x+2)}\right)$

$\ln\left(\frac{(x+2)(x+1)}{(x+1)^2(x+2)}\right)$

$\ln\left(\frac{1}{x+1}\right)$

7. Solve each equation. For non-integer answers, write your answer in exact form and then express your answer as a decimal rounded to two decimal places.

(a) $9^{x+2} = 27^{x-1}$ (5 points)

$(3^2)^{x+2} = (3^3)^{x-1}$

$3^{2(x+2)} = 3^{3(x-1)}$ same base

$2(x+2) = 3(x-1)$

$2x+4 = 3x-3$

$-2x \quad +3 \quad -2x \quad +3$

$7 = x$

(b) $3\log_2(6-2x) = 9$ (5 points)

$\frac{3\log_2(6-2x)}{3} = \frac{9}{3}$

$\log_2(6-2x) = 3$ Change from log form to exponential form

$2^3 = 6-2x$

$8 = 6-2x$

$-6 \quad -6$

$\frac{2}{-2} = \frac{-2x}{-2}$

$x = -1$

Check the inside of the log to make sure it's not negative!

$6-2(-1) = 6+2 = 8 =$ positive ✓

(c) $2^{3x+4} = 5$ (5 points)

$\ln 2^{3x+4} = \ln 5$

$(3x+4)\ln 2 = \ln 5$

$(3\ln 2)x + 4\ln 2 = \ln 5$

$-4\ln 2 \quad -4\ln 2$

$\frac{(3\ln 2)x}{3\ln 2} = \frac{\ln 5 - 4\ln 2}{3\ln 2}$

$x = \frac{\ln 5 - 4\ln 2}{3\ln 2}$

Note: Other answers are possible depending on which log was used

$x = -0.56$

Note: This will be the same regardless of which log was used

(d) $\log_8(x+6) = 1 - \log_8(x+4)$ (7 points)

$+\log_8(x+4) \quad +\log_8(x+4)$

$\log_8(x+6) + \log_8(x+4) = 1$

$\log_8((x+6)(x+4)) = 1$ Change from log form to exponential form

$(x+6)(x+4) = 8^1$

$x^2 + 10x + 24 = 8$

$-8 \quad -8$

$x^2 + 10x + 16 = 0$

$(x+8)(x+2) = 0$

$x = -8 \quad x = -2$

Check that the insides of the logs don't become negative:

$x = -8$: $-8+6 = -2$ and $-8+4 = -4$ ✗

$x = -2$: $-2+6 = 4$ and $-2+4 = 2$ ✓

$\boxed{x = -2}$

8. The number of milligrams of a drug in a patient's body t hours after the drug was administered is given by

$$D(t) = 50e^{-0.2t}.$$

(a) What is the initial amount of the drug given to the patient? (2 points)

$D(0) = 50e^{-0.2(0)} = 50e^{0} = 50(1) = 50$

(b) What is the amount of the drug in the patient's body after 4 hours? (2 points)

$D(4) = 50e^{-0.2(4)} = 50e^{-0.8} = 22.47$

(c) Suppose it's not safe for a doctor to administer another dose of the drug until there is only 5 milligrams of the drug left in the patient's body. What is the minimum amount of time after the initial dosage before the doctor can administer the next dose of the drug? Round your answer to the nearest hour. (6 points)

$\frac{5}{50} = \frac{50e^{-0.2t}}{50}$

$0.1 = e^{-0.2t}$ Change from exponential form to log form

$\frac{\ln 0.1}{-0.2} = \frac{-0.2t}{-0.2}$

$t = \frac{\ln 0.1}{-0.2} = 11.5 \rightarrow \boxed{t = 12}$

9. Solve each of the following systems of linear equations in two variables. (6 points each)

(a)

Note: There are more ways to solve this problem than just the ways I've shown

$$4x + y = -5$$
$$-x + 2y = 8$$

Substitution: $4x + y = -5$, $-4x \quad -4x$

$y = -4x - 5$

$-x + 2y = 8$

$-x + 2(-4x - 5) = 8$

$-x - 8x - 10 = 8$

$-9x - 10 = 8$

$+10 \quad +10$

$\frac{-9x}{-9} = \frac{18}{-9} \Rightarrow x = -2$

$y = -4(-2) - 5$

$= 8 - 5$

$= 3$

$(-2, 3)$

Addition: $4(-x + 2y = 8)$

$-4x + 8y = 32$

$+ \quad 4x + y = -5$

$9y = 27$

$\frac{9y}{9} = \frac{27}{9}$

$y = 3$

$-x + 2(3) = 8$

$-x + 6 = 8$

$-6 \quad -6$

$\frac{+x}{+1} = \frac{2}{-1}$

$x = -2$

$(-2, 3)$

(b)

Note: There are more ways to solve this problem than just the ways I've shown

$$8x - 10y = -24$$
$$12x - 8y = -8$$

Addition: $-3(8x - 10y = -24) \rightarrow -24x + 30y = 72$

$2(12x - 8y = -8) \rightarrow + \quad 24x - 16y = -16$

$14y = 56$

$\frac{14y}{14} = \frac{56}{14}$

$y = 4$

$8x - 10(4) = -24$

$8x - 40 = -24$

$+40 \quad +40$

$\frac{8x}{8} = \frac{16}{8}$

$x = 2$

$(2, 4)$

Bonus. Find the inverse of the following function, then verify that it is in fact the inverse. (10 points)

$$f(x) = 4^{x-1} + 3$$

$y = 4^{x-1} + 3$

$x = 4^{y-1} + 3$

$-3 \quad -3$

$x - 3 = 4^{y-1}$

Change from exponential form to log form

$\log_4(x-3) = y - 1$

$+1 \quad +1$

$1 + \log_4(x-3) = y$

$\boxed{f^{-1}(x) = 1 + \log_4(x-3)}$

$(f \circ f^{-1})(x) = f(1 + \log_4(x-3))$

$= 4^{1+\log_4(x-3)-1} + 3$

$= 4^{\log_4(x-3)} + 3$

$= x - 3 + 3$

$= x$ ✓

$(f^{-1} \circ f)(x) = f^{-1}(4^{x-1} + 3)$

$= 1 + \log_4(4^{x-1} + 3 - 3)$

$= 1 + \log_4 4^{x-1}$

$= 1 + x - 1$

$= x$ ✓

Extra Blank Graphs.