Section 2-4: Facts

For the following facts, suppose f is a polynomial function of degree 1 or higher of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0.$$

1. Rational Zeros Theorem: If a_n and a_0 are not zero and all the coefficients are integers, then if f has any rational zeros, they will be of the form $\frac{p}{q}$, where p is a factor of a_0 and q is a factor of a_n.

2. Linear Factorization Theorem: If $a_n \neq 0$, then f has exactly n complex zeros and can be written in the form

   $$f(x) = a_n(x - c_1)(x - c_2)\dots(x - c_n),$$

   where $c_1, c_2, \dots c_n$ are the n complex roots of f (some may be repeated).

   (a) Every polynomial function with real coefficients can be uniquely factored into a product of linear factors (that is, factors of the form $x - c$) and/or irreducible quadratic factors (that is, factors of the form $ax^2 + bx + c$, which cannot be factored).

3. Conjugate Zeros Theorem: If $a + bi$ $(b \neq 0)$ is a zero of f then its conjugate $a - bi$ is also a zero of f.

   (a) Note: A consequence of this theorem is that we can conclude that a polynomial function f of odd degree has at least one real zero.

4. Decartes' Rule of Signs: Assume that f is written in standard form; i.e., not factored and written in descending order of the exponents.

   (a) The number of positive real zeros of f either equals the number of sign changes of consecutive nonzero coefficients of $f(x)$ or else equals that number minus an even integer.

   (b) The number of negative real zeros of f either equals the number of sign changes of consecutive nonzero coefficients of $f(-x)$ or else equals that number minus an even integer.