Class Name : **Math 127 - Fall 2020 - 3007**

Student Name : Answer Key

Instructor Name : **Scheib**

Instructor Note :

Question 1 of 14

Verify that the equation is an identity.

Show that $\frac{1}{1+\cos a}+\frac{1}{1-\cos a}=2\csc^2 a.$

Question 2 of 14

Use an addition or subtraction formula to find the exact value.

Write your answer as a single fraction in simplest form. Rationalize your denominator, if necessary.

$$\sin\frac{7\pi}{12}$$

$$\sin\frac{7\pi}{12}=\square$$

Question 3 of 14

Find the exact value for the expression under the given conditions.

$\cos(\alpha-\beta)$, $\sin\alpha=\frac{1}{2}$ for α in Quadrant II and $\cos\beta=-\frac{1}{4}$ for β in Quadrant III.

$\cos(\alpha-\beta)=$ ☐

Question 4 of 14

Find the exact value. Write your answer as an integer or single fraction in simplest form. Rationalize your denominator, if necessary.

$$\sin\left[\arccos\left(-\frac{1}{2}\right)+\arcsin\left(\frac{\sqrt{2}}{2}\right)\right]$$

$\sin\left[\arccos\left(-\frac{1}{2}\right)+\arcsin\left(\frac{\sqrt{2}}{2}\right)\right]=$ ☐

Question 5 of 14

Verify the identity.

Show that $\frac{\cos(q+p)}{\sin q\,\cos p}=\cot q-\tan p$.

Question 6 of 14

Use the given information to find the exact function value. Simplify your answer as much as possible

$$\csc\theta = \frac{37}{12},\ \cos\theta > 0$$

(a) $\sin 2\theta$
(b) $\cos 2\theta$
(c) $\tan 2\theta$

Part 1 of 3

(a) $\sin 2\theta = \square$

Part 2 of 3

(b) $\cos 2\theta = \square$

Part 3 of 3

(c) $\tan 2\theta =$ ☐

Question 7 of 14

Verify the identity.

$$\cos^4 x - \sin^4 x = \cos 2x$$

Question 8 of 14

Use the given information to find the exact function value. Simplify your answer as much as possible. Rationalize the denominator if necessary.

$$\sin a = \frac{5}{13},\ 0 < a < \frac{\pi}{2}$$

(a) $\sin \frac{a}{2}$

(b) $\cos \frac{a}{2}$

(c) $\tan \frac{a}{2}$

Part 1 of 3

(a) $\sin \frac{a}{2} = \square$

Part 2 of 3

(b) $\cos \frac{a}{2} = \square$

Part 3 of 3

(c) $\tan \frac{a}{2} = \square$

Question 9 of 14

Write the product as a sum or difference.

$$\cos \frac{x}{4} \cos \frac{5x}{4}$$

$$\cos \frac{x}{4} \cos \frac{5x}{4} = \square$$

Question 10 of 14

Use a product-to-sum formula to find the exact value. Write your answer as a simplified fraction and rationalize the denominator, if necessary.

$$\cos \frac{25\pi}{24} \sin \frac{19\pi}{24}$$

$$\cos\frac{25\pi}{24}\sin\frac{19\pi}{24} = \square$$

Question 11 of 14

Write the expression as a product. Simplify your answer if necessary.

$$\cos 5b + \cos 6b$$

$$\cos 5b + \cos 6b = \square$$

Question 12 of 14

Use a sum-to-product formula to find the exact value. Write your answer as a simplified fraction and rationalize the denominator, if necessary.

$$\sin\frac{13\pi}{12} + \sin\frac{5\pi}{12}$$

$$\sin\frac{13\pi}{12} + \sin\frac{5\pi}{12} = \square$$

Question 13 of 14

Solve the equation. Write the numbers using integers or simplified fractions.

$$\sqrt{5}\cot\frac{x}{2} = \sqrt{5}$$

(a) Write the solution set for the general solution. Use n, where n is an integer.

(b) Write the solution set on the interval $[0, 2\pi)$.

Part 1 of 2

(a) Write the solution set for the general solution. Use n, where n is an integer.

The solution set for the general solution is $\{x \mid x = \square\}$.

Part 2 of 2

(b) Write the solution set on the interval $[0, 2\pi)$.

The solution set on the interval $[0, 2\pi)$ is $\{\square\}$.

Question 14 of 14

Solve the equation on the interval $[0, 2\pi)$. Write numbers as integers or simplified fractions and separate multiple answers with commas.

$$\sec^2\theta - 2 = 0$$

The solution set is $\{\square\}$.

1) $\frac{1}{1+\cos a} + \frac{1}{1-\cos a} = 2\csc^2 a$

$$\frac{1-\cos a}{(1+\cos a)(1-\cos a)} + \frac{1+\cos a}{(1-\cos a)(1+\cos a)}$$ LCD

$$\frac{1-\cancel{\cos a}+1+\cancel{\cos a}}{(1+\cos a)(1-\cos a)}$$ Add fractions

$$\frac{2}{(1+\cos a)(1-\cos a)}$$ Simplify

$$\frac{2}{1-\cancel{\cos a}+\cancel{\cos a}-\cos^2 a}$$ FOIL

$$\frac{2}{1-\cos^2 a}$$ Simplify

$$\frac{2}{\sin^2 a}$$ Pythagorean Identity

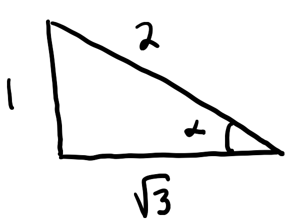
$$2\csc^2 a$$ Reciprocal Identity

Note: Other solutions possible

2) $\frac{7\cancel{\pi}}{\cancel{12}_1} \cdot \frac{\cancel{180°}^{15}}{\cancel{\pi}} = 75°$

$\sin \frac{7\pi}{12} = \sin 75° = \sin(30° + 45°) = \sin 30° \cos 45° + \cos 30° \sin 45°$

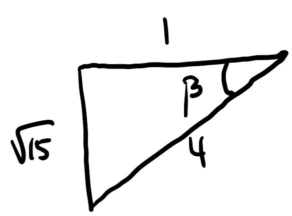
$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2}+\sqrt{6}}{4}$

3) 

$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

$b^2 = 3 \Rightarrow b = \sqrt{3}$



$1^2 + b^2 = 4^2$

$1 + b^2 = 16$

$b^2 = 15$

$b = \sqrt{15}$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$= \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{4}\right) + \frac{1}{2}\left(-\frac{\sqrt{15}}{4}\right) = \frac{\sqrt{3}}{8} - \frac{\sqrt{15}}{8} = \frac{\sqrt{3}-\sqrt{15}}{8}$$

4) $$\sin\left[\underbrace{\arccos\left(-\frac{1}{2}\right)}_{\frac{5\pi}{3}} + \underbrace{\arcsin\left(\frac{\sqrt{2}}{2}\right)}_{\frac{\pi}{4}}\right] = \sin\left(\frac{5\pi}{3}+\frac{\pi}{4}\right) = \sin\frac{5\pi}{3}\cos\frac{\pi}{4} + \cos\frac{5\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

5) $$\frac{\cos(q+p)}{\sin q\cos p} = \cot q - \tan p$$

Start: $$\frac{\cos(q+p)}{\sin q\cos p}$$

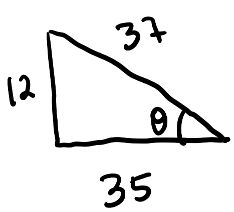
$$\frac{\cos q\cos p - \sin q\sin p}{\sin q\cos p}$$ Sum Formula

$$\frac{\cos q\cancel{\cos p}}{\sin q\cancel{\cos p}} - \frac{\cancel{\sin q}\sin p}{\cancel{\sin q}\cos p}$$ Separate into two fractions

$$\frac{\cos q}{\sin q} - \frac{\sin p}{\cos p}$$ Simplify

$$\cot q - \tan p$$ Quotient Identity

Note: Other solutions possible

6) 

$$a^2 + 12^2 = 37^2$$

$$a^2 + 144 = 1369$$

$$a^2 = 1225$$

$$a = 35$$

$\csc\theta > 0$ & $\cos\theta > 0 \Rightarrow$ Quad I

a) $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{12}{37}\right)\left(\frac{35}{37}\right) = \frac{840}{1369}$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(\frac{35}{37}\right)^2 - \left(\frac{12}{37}\right)^2 = \frac{1225}{1369} - \frac{144}{1369} = \frac{1081}{1369}$

Note: Either of the other double angle formulas for cosine could have been used instead

c) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{12}{35}\right)}{1-\left(\frac{12}{35}\right)^2} = \frac{\frac{24}{35}\cdot 1225}{\left(1-\frac{144}{1225}\right)\cdot 1225} = \frac{840}{1225-144} = \frac{840}{1369}$

7) $\cos^4 x - \sin^4 x = \cos 2x$

Start: $\cos^4 x - \sin^4 x$

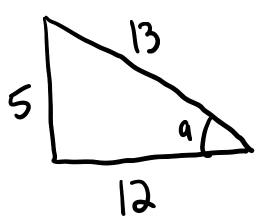
$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$ — Factor (difference of squares)

$(1)(\cos^2 x - \sin^2 x)$ — Pythagorean Identity

$\cos^2 x - \sin^2 x$ — Multiply

$\cos 2x$ — Double-angle Formula

8)



$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

$0 < a < \frac{\pi}{2} \Rightarrow \frac{0}{2} < \frac{a}{2} < \frac{\frac{\pi}{2}}{2}$

$0 < \frac{a}{2} < \frac{\pi}{4}$

$\frac{a}{2}$ in Quad I

a) $\sin\frac{a}{2} = \underset{\text{Quad I}}{\pm}\sqrt{\frac{1-\cos a}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}} = \sqrt{\frac{1}{26}} = \frac{\sqrt{1}}{\sqrt{26}}$

$= \frac{1}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \frac{\sqrt{26}}{26}$

b) $\cos\frac{a}{2} = \underset{\text{Quad I}}{\pm}\sqrt{\frac{1+\cos a}{2}} = \sqrt{\frac{1+\frac{12}{13}}{2}} = \sqrt{\frac{\frac{13}{13}+\frac{12}{13}}{2}} = \sqrt{\frac{\frac{25}{13}}{2}} = \sqrt{\frac{25}{26}} = \frac{5\cdot\sqrt{26}}{\sqrt{26}\cdot\sqrt{26}}$

$= \frac{5\sqrt{26}}{26}$

c) $\tan\frac{a}{2} = \frac{1-\cos a}{\sin a} = \frac{\left(1-\frac{12}{13}\right)\cdot 13}{\frac{5}{13}\cdot 13} = \frac{13-12}{5} = \frac{1}{5}$

9) $\cos\frac{x}{4}\cos\frac{5x}{4} = \frac{1}{2}\left[\cos\left(\frac{x}{4}-\frac{5x}{4}\right)+\cos\left(\frac{x}{4}+\frac{5x}{4}\right)\right] = \frac{1}{2}\left[\cos(-x)+\cos\left(\frac{3x}{2}\right)\right]$

$= \frac{1}{2}\left[\cos x + \cos\left(\frac{3x}{2}\right)\right]$

10) $\cos\frac{25\pi}{24}\sin\frac{19\pi}{24} = \frac{1}{2}\left[\sin\left(\frac{25\pi}{24}+\frac{19\pi}{24}\right)+\sin\left(\frac{25\pi}{24}-\frac{19\pi}{24}\right)\right] = \frac{1}{2}\left[\sin\left(\frac{44\pi}{24}\right)+\sin\left(\frac{6\pi}{24}\right)\right]$

$= \frac{1}{2}\left[\sin\frac{11\pi}{6}+\sin\frac{\pi}{4}\right] = \frac{1}{2}\left(-\frac{1}{2}+\frac{\sqrt{2}}{2}\right) = \frac{1}{2}\left(\frac{-1+\sqrt{2}}{2}\right) = \frac{-1+\sqrt{2}}{4}$

11) $\cos 5b + \cos 6b = 2\cos\left(\frac{5b+6b}{2}\right)\cos\left(\frac{5b-6b}{2}\right) = 2\cos\left(\frac{11b}{2}\right)\cos\left(-\frac{b}{2}\right) = 2\cos\left(\frac{11b}{2}\right)\cos\left(\frac{b}{2}\right)$

12) $\sin\frac{13\pi}{12}+\sin\frac{5\pi}{12} = 2\sin\left(\frac{\frac{13\pi}{12}+\frac{5\pi}{12}}{2}\right)\cos\left(\frac{\frac{13\pi}{12}-\frac{5\pi}{12}}{2}\right) = 2\sin\left(\frac{\frac{3\pi}{2}}{2}\right)\cos\left(\frac{\frac{2\pi}{3}}{2}\right)$

$= 2\sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{2}$

13) $\frac{\sqrt{5}\cot\frac{x}{2}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}} \Rightarrow \cot\frac{x}{2} = 1 \Rightarrow 2\cdot\frac{x}{2} = \cot^{-1}(1)\cdot 2 \Rightarrow x = 2\cot^{-1}(1)$

a) $x = 2\cot^{-1}(1) \Rightarrow x = 2\left(\frac{\pi}{4}+k\pi\right) \Rightarrow x = \frac{\pi}{2}+2k\pi,\ k\in\mathbb{Z}$

b) $k=0$: $x = \frac{\pi}{2}$

14) $\sec^2\theta - 2 = 0$

$\quad +2 \quad +2$

$\sec^2\theta = 2 \Rightarrow \sec\theta = \pm\sqrt{2} \Rightarrow \cos\theta = \pm\frac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{2}}{2} \Rightarrow \cos\theta = \frac{\sqrt{2}}{2}$ or $\cos\theta = -\frac{\sqrt{2}}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ or $\theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{7\pi}{4}$ or $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$