

## Trigonometric Identities

### Sum and Difference Formulas

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

### Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta}$$

$$\tan \frac{\theta}{2} = \frac{\sin \theta}{1 + \cos \theta}$$

### Double-Angle Formulas

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = 2 \cos^2 \theta - 1$$

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

### Product-to-Sum Formulas

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x - y) + \cos(x + y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

### Sum-to-Product Formulas

$$\sin x + \sin y = 2 \sin \left( \frac{x+y}{2} \right) \cos \left( \frac{x-y}{2} \right)$$

$$\sin x - \sin y = 2 \sin \left( \frac{x-y}{2} \right) \cos \left( \frac{x+y}{2} \right)$$

$$\cos x + \cos y = 2 \cos \left( \frac{x+y}{2} \right) \cos \left( \frac{x-y}{2} \right)$$

$$\cos x - \cos y = -2 \sin \left( \frac{x+y}{2} \right) \sin \left( \frac{x-y}{2} \right)$$